GCE 2004

June Series

ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark Scheme

Mathematics and Statistics B

MBP3

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Key to Mark Scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m marks and is for	accuracy
B	mark is independent of M or m marks and is for	accuracy
E	mark is for	explanation
√or ft or F		follow through from previous incorrect result
cao		correct answer only
cso		correct solution only
awfw		anything which falls within
awrt		anything which rounds to
acf		any correct form
ag		answer given
sc		special case
oe		or equivalent
sf		significant figure(s)
dp		decimal place(s)
A2,1		2 or 1 (or 0) accuracy marks
–*x* ee		deduct *x* marks for each error
pi		possibly implied
sca		substantially correct approach

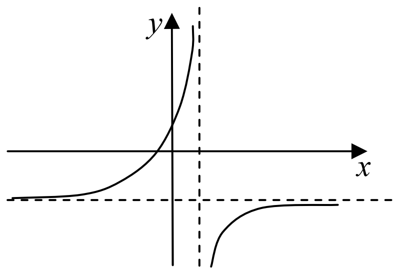
Abbreviations used in Marking

MC – *x*	deducted *x* marks for mis-copy
MR – *x*	deducted *x* marks for mis-read
isw	ignored subsequent working
bod	given benefit of doubt
wr	work replaced by candidate
fb	formulae book

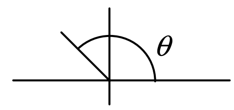
Application of Mark Scheme

No method shown:	
Correct answer without working	**mark as in scheme**
Incorrect answer without working	**zero marks unless specified otherwise**
More than one method / choice of solution:	
2 or more complete attempts, neither/none crossed out	**mark both/all fully and award the mean mark rounded down**
1 complete and 1 partial attempt, neither crossed out	**award credit for the complete solution only**
Crossed out work	**do not mark unless it has not been replaced**
Alternative solution **using a correct or partially correct method**	**award method and accuracy marks as appropriate**

Mathematics and Statistics B Pure 3 MBP3 June 2004

Question Number and Part	Solution	Marks	Total	Comments
1(a)	$A^{-1} = \frac{1}{\det A}\begin{bmatrix}1 & 5\\ -4 & 3\end{bmatrix}$	M1		Condone one slip in matrix, multiplication by det A, or omission of det A
	$= \frac{1}{23}\begin{bmatrix}1 & 5\\ -4 & 3\end{bmatrix}$	A1	2	Any equivalent
(b)	$\begin{bmatrix}x\\ y\end{bmatrix} = \frac{1}{23}\begin{bmatrix}1 & 5\\ -4 & 3\end{bmatrix}\begin{bmatrix}11\\ 7\end{bmatrix}$	M1		Must premultiply by A^{-1}
		A1√		Either x or y ft their inverse
	$x = 2, \quad y = -1$	A1	3	Both correct from correct inverse matrix
	Total		**5**	
2(a)(i)	(0,4) and	B1		
	$\left(-\frac{4}{3}, 0\right)$	B1	2	
(ii)	Asymptote at $x = \frac{1}{2}$ and at	B1		
	$y = -1\frac{1}{2}$	B1	2	
(iii)		M1		One branch roughly correct
		A1	2	Good graph
(b)	$3x + 4 = 1 - 2x \quad \Rightarrow 5x = -3$	M1		
	$\Rightarrow x = -\frac{3}{5}$	A1	2	
(c)	Use of value from (b)	M1		If algebraic method – must be sound eg simply multiplying up to give $3x + 4 \le 1 - 2x \Rightarrow$ M0
	$\Rightarrow x \le -\frac{3}{5}$	A1		
	Also $x > \frac{1}{2}$	B1	3	
	Total		**11**	
3(a)(i)	$\alpha + \beta = -(7 + p)$	B1		
	$\alpha\beta = p$	B1	2	
(b)	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$	M1		
	$= (7 + p)^2 - 2p$	A1	2	oe $p^2 + 12p + 49$
(c)(i)	$(\alpha - \beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta$	M1		
	$= p^2 + 12p + 49 - 2p = p^2 + 10p + 49$	A1	2	**ag**
(ii)	$(\alpha - \beta)^2 = 25$	B1		
	$p^2 + 10p + 49 = 25 \quad \Rightarrow p^2 + 10p + 24 = 0$	M1		May be using 5 etc instead of 25
	$p = -4, \; p = -6$	A1	3	
	Total		**9**	

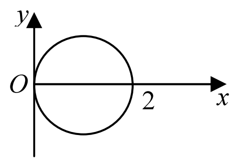
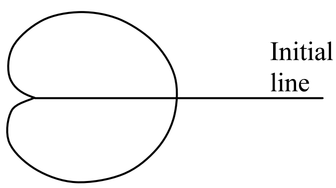
MBP3 (cont)

Question Number and Part	Solution	Marks	Total	Comments
4(a)	$\lvert -1+\sqrt{3}\mathrm{i} \rvert = \sqrt{(1+3)}$	M1		
	$= 2$	A1		
	$\tan^{-1}\left(\sqrt{3}\right) = \dfrac{\pi}{3}$	M1		Use of $\tan^{-1}\left(\dfrac{y}{x}\right)$  Or sketch
	Argument $= \dfrac{2\pi}{3}$	A1	4	$120°$ or $2.094395\ldots$without working earns M1 , A0
(b)	$\left(-1+\sqrt{3}\mathrm{i}\right)^2 = 1-3-2\sqrt{3}\mathrm{i}$	M1		3 term attempt at square or binomial for cubic with terms using 1 3 3 1
	$\left(-2-2\sqrt{3}\mathrm{i}\right)\left(-1+\sqrt{3}\mathrm{i}\right) =$	m1		Or simplifying individual terms of cubic
	$2+6+2\sqrt{3}\,\mathrm{i}-2\sqrt{3}\,\mathrm{i}=8$	A1	3	Use of DeMoivre; (modulus cubed), arg mutiplied by 3 (M2), final ans A1
(c)(i)	$k=-8$	B1√	1	ft their real value in (b)
(ii)	$-1-\sqrt{3}\mathrm{i}$ is other complex root	B1	1	
	Total		**9**	
5(a)	$\boldsymbol{AB} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$	M1 A1	2	At least two entries correct All correct
(b)(i)	Reflection	M1		
	in $y=x$	A1	2	
(ii)	Reflection	M1		
	in x-axis	A1	2	
(iii)	Rotation (about origin)	M1		
	through $\dfrac{\pi}{2}$ (anticlockwise)	A1	2	
	Total		**8**	

MBP3 (cont)

Question Number and Part	Solution	Marks	Total	Comments
6(a)	ln3 = 1.0986... $\ln y = 1.33$ $y = 3.8$	M1 m1 A1	 3	 Condone 1.30 to 1.35 Accept 3.7 to 3.9
(b)(i)	$\ln y = \ln A + n \ln x$	B1	1	
(ii)	$\ln A = 0.80$ (intercept on ln y-axis) $A = 2.2$ n = gradient of line $= 0.48$	M1 A1 M1 A1	 4	 Condone value rounding to this Accept value rounding to 0.47, 0.48 or 0.49
	Total		**8**	
7(a)	$\dfrac{4-4(k+3)}{(k+2)(k+3)}$ $=\dfrac{-4(k+2)}{(k+2)(k+3)}=\dfrac{-4}{(k+3)}$	M1 A1	 2	 **ag** be convinced
(b)	When $n=1$; RHS $=2-\dfrac{4}{3}=\dfrac{2}{3}$; LHS $=\dfrac{2}{3}$ Assume formula true for $n=k$ Add $(k+1)$th term to both sides namely $\dfrac{4}{(k+2)(k+3)}$ RHS $=2-\dfrac{4}{(k+2)}+\dfrac{4}{(k+2)(k+3)}$ $=2-\dfrac{4}{(k+3)}$ Result true for $n = k + 1$ Hence true for n = 1, 2, 3 etc by induction	B1 E1 M1 A1	 4	(True when $n=1$) Plus the conclusion; hence true …
(c)(i)	$u_1=\dfrac{2}{3}$; $u_2=\dfrac{1}{3}$ Hence sum $=1-\dfrac{4}{(n+2)}$	M1 A1	 2	 Condone N or r instead of n
(ii)	Sum to infinity = 1	B1√	1	ft their (c)(i)
	Total		**9**	

MBP3 (cont)

Question Number and Part	Solution	Marks	Total	Comments
8(a)(i)	Translation	M1		
	through $\begin{bmatrix}1\\0\end{bmatrix}$	A1	2	Allow M1, A0 for wrong term (eg shift, move) but correct vector
(ii)		M1 A1	2	circle Centre (1,0) , radius 1 & through (0,0)
(b)	$x^2 + y^2 = r^2$, $x = r\cos\theta$	B1		Either seen
	$x^2 + y^2 - 2x = 2\sqrt{x^2 + y^2}$			
	$r^2 - 2r\cos\theta = 2r$	M1		Attempt to sub in cartesian equation or polar equation $x = r\cos\theta$ **&** $x^2 + y^2 = r^2$
	$\Rightarrow r = 2 + 2\cos\theta$	A1	3	**ag** be convinced
(c)(i)	Greatest value of $r = 4$ Least value of $r = 0$	B1 B1	2	
(ii)		B1 B1 B1	3	Correct one quadrant Symmetry about initial line Good graph
	Total		**12**	
9(a)	Yes, closed $a, b \in \mathbb{Z} \Rightarrow a + b - 4 \in \mathbb{Z}$	B1 E1	2	Explanation showing understanding of integers and closure
(b)	$a \otimes e = a$ or $e \otimes a = a$ $\Rightarrow a + e - 4 = a \quad \Rightarrow e = 4$	M1 A1	2	**sc** 1 only if found from table of values
(c)	$a \otimes x = e$ or $x \otimes a = e$ $\Rightarrow a + x - 4 = 4$ $\Rightarrow a = 8 - x$	M1 A1	2	Full marks for correct answer
(d)	$(a \otimes b) \otimes c = (a + b - 4) + c - 4$ $= a + b + c - 8$	B1		Or $a \otimes (b \otimes c)$ correct
	Considering $(a \otimes b) \otimes c$ and $a \otimes (b \otimes c)$	M1		
	Shown to be equal	A1	3	A0 if $\otimes$ assumed to be commutative
	Total		**9**	
	TOTAL		**80**	