GCE 2005
January Series

ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark Scheme

Mathematics and Statistics B
(MBP3)

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Key to Mark Scheme

M ..... mark is for ..... method
m ..... mark is dependent on one or more M marks and is for ..... method
A ..... mark is dependent on M or m marks and is for ..... accuracy
B ..... mark is independent of M or m marks and is for ..... method and accuracy
E ..... mark is for ..... explanation
√ **or ft or F** ..... follow through from previous incorrect result
CAO ..... correct answer only
AWFW ..... anything which falls within
AWRT ..... anything which rounds to
AG ..... answer given
SC ..... special case
OE ..... or equivalent
A2,1 ..... 2 or 1 (or 0) accuracy marks
-*x* EE ..... deduct *x* marks for each error
NMS ..... no method shown
PI ..... possibly implied
SCA ..... substantially correct approach
c ..... candidate
SF ..... significant figure(s)
DP ..... decimal place(s)

Abbreviations used in Marking

MC – *x* ..... deducted *x* marks for mis-copy
MR – *x* ..... deducted *x* marks for mis-read
ISW ..... ignored subsequent working
BOD ..... given benefit of doubt
WR ..... work replaced by candidate
FB ..... formulae booklet

Application of Mark Scheme

No method shown:
Correct answer without working ..... mark as in scheme
Incorrect answer without working ..... zero marks unless specified otherwise

More than one method/choice of solution:

2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only

Crossed out work — do not mark unless it has not been replaced

Alternative solution using a correct or partially correct method — award method and accuracy marks as appropriate

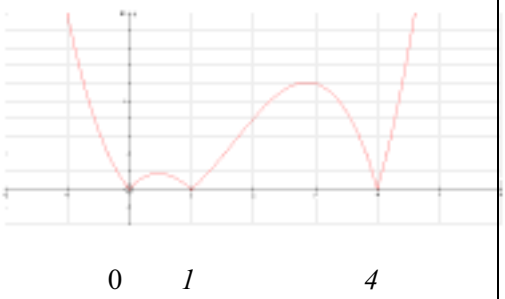
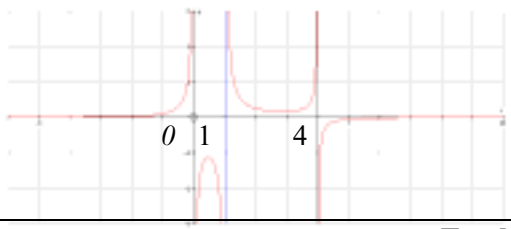
Mathematics and Statistics B Pure 3 MBP3 January 2005

Question Number and Part	Solution	Marks	Total	Comments
1(a)	$z = 3\sqrt{2}\left(\cos\frac{3\pi}{4} + \mathrm{i}\sin\frac{3\pi}{4}\right) = -3 + 3\mathrm{i}$	M1 A1	2	Give M1 if either a, b correct
(b)(i)	$w^2 = 1 - 3 - 2\mathrm{i}\sqrt{3}$	M1		
	so that $w^2 + 2w = -4 \in \mathbb{R}$	A1	2	
(ii)	$\frac{4}{w} = \frac{4}{-1+\mathrm{i}\sqrt{3}} \times \frac{-1-\mathrm{i}\sqrt{3}}{-1-\mathrm{i}\sqrt{3}}$	M1		
	$= -1 - \mathrm{i}\sqrt{3}$	A1		
	$w - \frac{4}{w} = 2\mathrm{i}\sqrt{3}$	A1	3	cao
	Total		**7**	
2	Multiplying by $(3-x)^2$	M1		
	$(3-x)\{3x + 1 - 2(3-x)\} > 0$	m1		Collecting up on one side
	$5(x-1)(x-3) < 0$	B1√		$x = 1, 3$ identified ft
	$1 < x < 3$	A1	4	cao
	ALTERNATIVE 1:			
	For $x < 3$, $3x + 1 > 2(3-x)$	M1		≥ 1 case correctly considered
	$\Rightarrow x > 1$	A1		
	For $x > 3$, $3x + 1 < 2(3-x)$	M1		
	$\Rightarrow x < 1 \Rightarrow$ no solns.	A1	(4)	Must have a definite conclusion
	ALTERNATIVE 2:			
	Relevant graph drawn	M1 A1		Ignore irrelevant y-values
	Identifying correct intersections	B1√		ft if appropriate
	Correct range deduced	A1	(4)	
	Total		**4**	
3(a)(i)	$\mathbf{M}^2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$	M1		Must be evidence of correct matrix multn. method
	$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$	A1		cao
(ii)	$\mathbf{M}^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	B1√	3	ft
(b)	Rotation	M1		
	(anticlockwise) through ½π about 0	A1	2	"acw" may be implicit
(c)	$\mathbf{N} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$	M1 A1	2	Any suitable method made clear
	Total		**7**	

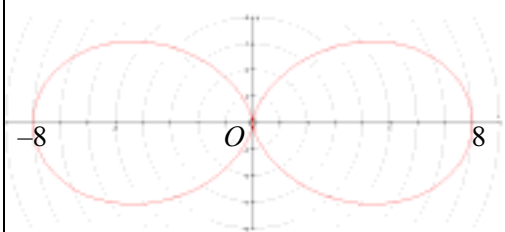
MBP3 (cont)

Question Number and Part	Solution	Marks	Total	Comments
4(a)(i)	$\alpha + \beta = 2$, $\alpha\beta = ½$	B1 B1	2	
(ii)	$(\alpha+\beta)^2 - 2\alpha\beta = 3$	M1 A1√	2	ft (i)'s answers
(iii)	$\alpha^2 + \beta^2 + 6(\alpha+\beta) + 18 = 33$	M1 A1√	2	ft (i) and (ii)'s answers
(b)	New product of roots = 1	B1		
	New sum of roots = $\dfrac{(\alpha+3)^2+(\beta+3)^2}{\alpha\beta+3(\alpha+\beta)+9}$	M1		Form ready for substn.
	$= \dfrac{66}{31}$	A1√		ft
	New eqn. is $31y^2 - 66y + 31 = 0$	A1√	4	ft. Must have integer coefficients and be an equation (coefft. $y^2 \neq 1$)
	Total		**10**	
5(a)	$\ln y = \ln a + x \ln b$	B1	1	
(b)(i)	$\ln y$ 1.128 1.261 1.394 1.528 1.660	B1 B1		3 roots (to ≥ 3 s.f.) All roots to 3 d.p. (condone 1.66)
	Points plotted on graph provided	B1	3	Reasonably accurately
(ii)	"Good" straight line drawn	B1	1	
(c)(i)	From graph $x = 3.4 \Rightarrow \ln y = 1.44/5$ $\Rightarrow y = 4.24$ to 4.26	M1 A1	2	Including un-logging attempt awrt
(ii)	Method for finding gradient: $\ln b \approx \dfrac{0.67}{0.5} \approx 1.32\text{-}4$	M1		Sim. Eqns. Approach OK also For either/both M's
	$b = 3.7\text{-}3.9$	A1		awrt
	Reading off y-intercept: $\ln a \approx 0.99$	M1		
	$a = 2.7$	A1	4	awrt
	Total		**11**	

MBP3 (cont)

Question Number and Part	Solution	Marks	Total	Comments
6(a)		M1 A1	2	All above x-axis Good graph, with cusps. Ignore vertical scale.
(b)	$x = 0$, $x = 1$, $x = 4$, $y = 0$ 	B1 B1 B1 B1 B1	5	Any 2 asymptotes stated All 4 Region $0 < x < 1$ correct Region $1 < x < 4$ correct Regions $x < 0$, $x > 4$ correct
	Total		**7**	
7(a)	$a = 11$, $b = 9$, $c = 3$, $d = 1$	B1 B1	2	2 roots ; all 4 roots
(b)	$k = 9$	B1	1	
(c)	$3^{-1} = 5$	B1	1	
(d)	$x = 11$	B1	1	
(e)	$k^2 = 81 \equiv 11 \pmod{14} \equiv k + 2$	M1 A1	2	ft M only for k^2 (mod 14) correct
	Total		**7**	

MBP3 (cont)

Question Number and Part	Solution	Marks	Total	Comments
8(a)(i)	Translation (// x-axis), vector $\begin{bmatrix}2\\0\end{bmatrix}$	M1 A1	2	B1 for equivalent correct description without "translation"
(ii)	$(r\cos\theta - 2)^2 + (r\sin\theta)^2 = 4$ $r^2(\cos^2\theta + \sin^2\theta) - 4r\cos\theta + 4 = 4$ Use of $c^2 + s^2 = 1$ $(r \neq 0) \Rightarrow r = 4\cos\theta$	M1 A1 B1 A1	4	Backwards approach is fine **ag**
(b)(i)	$r_{max} = 8$, $r_{min} = 0$	B1 B1	2	
(ii)		B1 B1 B1	3	Symmetry in $\theta = \frac{1}{2}\pi$ Symmetry in $\theta = 0$ All correct
(c)	Equating $8\cos^2\theta = 4\cos\theta$ and solving $\theta = \frac{1}{3}\pi$ and $r = 2$ 2nd point $\theta = -\frac{1}{3}\pi$, $r = 2$	M1 A1 A1 A1√	4	Or ft $2\pi - (1^{st}\ \theta)$, same r
	Total		**15**	

MBP3 (cont)

Question Number and Part	Solution	Marks	Total	Comments
9(a)	For $n = 1$, LHS = RHS = 96	B1		True case $n = 1$
	Clear induction hypothesis somewhere	E1		
	Correct $(k + 1)^{th}$ term used	B1		$4(k + 2)(k + 3)(k + 4)$
	Some $(k + 1)^{th}$ term added both sides	M1		
	$(k + 2)(k + 3)(k + 4)\{k + 1 + 4\} - 24$	m1		Factorising attempt
	$=[(k+1)+1][(k+1)+2][(k+1)+3]...$ $...[(k+1)+4]-24$			
	Or explaining that formula true for $n = k \Rightarrow$ true also for $n = k + 1$	A1	6	Convincingly
(b)(i)	$\frac{r+3-(r+1)}{(r+1)(r+2)(r+3)} \equiv \frac{2}{(r+1)(r+2)(r+3)}$	B1	1	Shown
	$\sum \frac{2}{(r+1)(r+2)(r+3)} =$ $\sum \frac{1}{(r+1)(r+2)} - \sum \frac{1}{(r+1)(r+2)}$	M1		Attempt at difference of two series
	$= \left\{\frac{1}{2.3}+\frac{1}{3.4}+...+\frac{1}{(n+1)(n+2)}\right\} -$ $\left\{\frac{1}{3.4}+...+\frac{1}{(n+1)(n+2)}+\frac{1}{(n+2)(n+3)}\right\}$	A1		Correct series identified
	$= \frac{1}{2.3} - \frac{1}{(n+2)(n+3)}$	m1		All terms except 1st and last cancelling; $A = \frac{1}{6}$
		A1	4	Give B1 if A deduced correctly
(iii)	$S = \frac{1}{6}$	B1√	1	ft their A
	Total		**12**	
	TOTAL		**80**	