AQA
ASSESSMENT and QUALIFICATIONS ALLIANCE

# Mark scheme January 2004

# GCE

## Mathematics & Statistics B

## Unit MBP3

# Key to mark scheme

| | | |
|---|---|---|
| **M** | mark is for | method |
| **m** | mark is dependent on one or more M marks and is for | method |
| **A** | mark is dependent on M or m mark and is for | accuracy |
| **B** | mark is independent of M or m marks and is for | method and accuracy |
| **E** | mark is for | explanation |
| **√ or ft or F** | | follow through from previous incorrect result |
| **CAO** | | correct answer only |
| **AWFW** | | anything which falls within |
| **AWRT** | | anything which rounds to |
| **AG** | | answer given |
| **SC** | | special case |
| **OE** | | or equivalent |
| **A2,1** | | 2 or 1 (or 0) accuracy marks |
| **– $x$ EE** | | Deduct $x$ marks for each error |
| **NMS** | | No method shown |
| **PI** | | Perhaps implied |
| **c** | | Candidate |

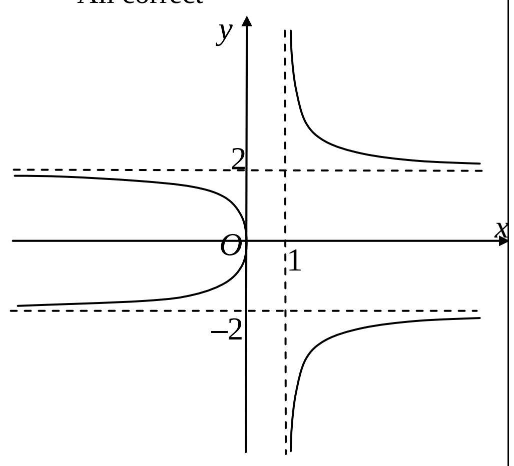
# Abbreviations used in marking

| | |
|---|---|
| **MC – $x$** | deducted $x$ marks for miscopy |
| **MR – $x$** | deducted $x$ marks for misread |
| **ISW** | ignored subsequent working |
| **BOD** | gave benefit of doubt |
| **WR** | work replaced by candidate |

# Application of mark scheme

| | |
|---|---|
| Correct answer without working | mark as in scheme |
| Incorrect answer without working | zero marks unless specified otherwise |

Award method and accuracy marks as appropriate to an alternative solution using a correct method or partially correct method.

| Question number and part | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 1(a)(i) | $\alpha + \beta = -2$ , $\alpha\beta = 3$ | B1 B1 | 2 | |
| (ii) | $(\alpha + \beta)^3 - 3\,\alpha\beta(\alpha + \beta)$ | M1 A1 | | Or $(\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$ & $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ |
| | $\Rightarrow \alpha^3 + \beta^3 = 10$ | A1 | 3 | **ag** |
| (iii) | $\dfrac{\alpha^3 + \beta^3}{(\alpha\beta)^3} = \dfrac{10}{27}$ | M1 A1 | 2 | |
| (b) | New product of roots = $\dfrac{1}{(\alpha\beta)^3} = \dfrac{1}{27}$ | B1 | | |
| | $x^2$ – [cand's (a) (iii)] $x$ + [cand's product] | M1 | | |
| | $\Rightarrow\ 27x^2 - 10\,x + 1 = 0$ | A1√ | 3 | ft Must have integer coefficients and be an equation |
| | **Total** | | **10** | |
| 2(a) | $\subset$ - shaped parabola | M1 | | |
| | Vertex at $O$, good sketch, symmetry obvious | A1 | 2 | *Essentially* all correct |
| (b) | $x^2 = 8\,y$ or equivalent | M1 A1 | 2 | M1 for general idea |
| (c) | Translation; by vector $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$ | M1 A1 | 2 | **sc**: B1 for correct description without "translation" |
| | **Total** | | **6** | |
| 3(a) | $a = 4$ and $b = 1$ | B1 B1 | 2 | |
| (b) | Asymptotes $x = 1$ , $y = 2$ , $y = -2$ | B1 B1 | | One correct; second correct |
| | Graph: Correct for $y > 0$ | B1 | | |
| | Symmetry in $x$-axis | B1 | | |
| | All correct | B1 | 5 | Or B1 for each correct region |
| |  | | | E.g. 4/5 for all correct graph but with asymptotes $x = 1$, $y = \pm 4$ |
| | **Total** | | **7** | |

| Question number and part | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 4(a) | $24 - 3k$ | B1 | 1 | |
| (b) | Det = 0 $\Rightarrow$ $k = 8$ | M1√<br>A1√ | 2 | ft (a) |
| (c)(i) | Area = 0 | B1√ | 1 | ft $5 \times$ cand's Det with $k = 8$ |
| (ii) | Det = 3 and / or $-3$<br>$\Rightarrow$ $k = 7$ $\Rightarrow$ $k = 9$ | M1<br>A1√<br>A1 | 3 | ft cand's "$24 - 3k = 3$"<br>cao |
| | **Total** | | **7** | |
| 5(a) | $\ln Q = \ln a + b \ln x$ | B1 | 1 | |
| (b)(i) | $\ln x$ : –0.92 –0.69 –0.51 –0.36 –0.22<br>$\ln Q$ : 0.54 1.11 1.56 1.94 2.28<br>Points plotted on graph provided | B1<br>B1<br>B1 | 3 | Most correct<br>At most one error<br>Reasonably accurately |
| (ii) | "Good" line of best fit drawn | B1 | 1 | |
| (c)(i) | $\ln Q = 1.29 - 1.30$ $\Rightarrow$ $Q \approx 3.6 - 3.7$ | M1 A1 | 2 | |
| (ii) | Method for finding gradient: $b = 2.5$<br>Reading off $y$-intercept: $\ln a \approx 2.8$<br>$a = 16 - 17$ | M1 A1<br>M1<br>A1 | 4 | $\pm 0.1$<br>Give M marks for simultaneous equations approach |
| | **Total** | | **11** | |
| 6(a)(i) | $-5 + 12\,\mathrm{i}$ | M1 A1 | 2 | |
| (ii) | Squaring their answer to (i) or use of the binomial theorem: $-119 - 120\,\mathrm{i}$ | M1<br>A1√ | 2 | ft |
| (b)(i) | Subst$^{g}$. their $z^4$, $z = 2 + 3\,\mathrm{i}$ into equation<br>$(-119 - 120\,\mathrm{i}) + 40(2 + 3\,\mathrm{i}) + k = 0$<br>$\Rightarrow$ $k = 39$ | M1<br>A1 | 2 | cao |
| (ii) | $2 - 3\,\mathrm{i}$ | B1 | 1 | Or $z = -1, -3$ |
| | **Total** | | **7** | |
| 7(a)(i) | 8 6 4 2<br>6 8 10 12<br>4 10 2 8<br>2 12 8 4 | B1<br>B1<br>B1<br>B1 | 4 | One for each correct row/column |
| (ii) | Only elements of $S$ appear in the Cayley table | E1 | 1 | Or equivalent statements |
| (iii) | The identity is 8 | B1 | 1 | |
| (iv) | $12^{-1} = 10$ | B1 | 1 | |
| (b) | $x \equiv 6 \pmod{14}$ but allow $x = 6$ | B1 | 1 | |
| (c) | $x = 4$ and $x = 10$ | B1 B1 | 2 | **sc** B1 for $x^2 \equiv 2 \pmod{14}$ only |
| | **Total** | | **10** | |

| Question number and part | Solution | Marks | Total | Comments |
| --- | --- | --- | --- | --- |
| 8(a) | $r_{max} = 2$ when $\theta = \frac{1}{4}\pi$ and $\theta = -\frac{3}{4}\pi$ | B1 B1<br>B1 | 3 | |
| (b) | $\sin 2\theta = -1 \Rightarrow 2\theta = \frac{3}{2}\pi, \ldots$ | M1 A1 | | |
| | giving $\theta = \frac{3}{4}\pi$, $\theta = -\frac{1}{4}\pi$ | A1 A1 | 4 | Penalise degrees max. once; ignore correct out-of-range answers |
| (c) | Use of $r = \sqrt{x^2 + y^2}$ | B1 | | |
| | Use of either $\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$ | M1 | | |
| | $\sqrt{x^2 + y^2} = 1 + \frac{2xy}{x^2 + y^2}$ | A1 | 3 | Any correct form at earliest stage |
| | **Total** | | **10** | |
| 9(a)(i) | Attempt at $f(r + 1) - f(r)$<br>$= r(r + 1)(r + 2)(r + 3)$<br>$- (r - 1)r(r + 1)(r + 2)$<br>$= r(r + 1)(r + 2)\{ r + 3 - r + 1\}$<br>$= 4r(r + 1)(r + 2)$ | M1<br><br><br><br>A1 | 2 | i.e $k = 4$ |
| (ii) | $\sum r(r+1)(r+2) = \frac{1}{4}\sum\{f(r+1) - f(r)\}$ | M1√ | | ft $k$ |
| | $= \frac{1}{4}\{f(n+1) - f(1)\}$ | m1 | | |
| | $= \frac{1}{4}n(n+1)(n+2)(n+3)$ | A1 | 3 | |
| (b) | For $n = 1$, LHS = RHS = $\frac{1}{3}$ | B1 | | |
| | Adding next term to at least the RHS | M1 | | |
| | Correct $(k + 1)^{th}$ term used: $\frac{2}{(k+1)(k+2)(k+3)}$ | B1 | | |
| | RHS = $\frac{1}{2} - \left\{\frac{(k+3)-2}{(k+1)(k+2)(k+3)}\right\}$ | M1 | | With correct sign |
| | $= \frac{1}{2} - \frac{1}{(k+2)(k+3)}$ | A1 | | |
| | Clear induction hypothesis somewhere | E1 | 6 | Or full explanation at end |
| (c) | $S = \frac{1}{2}$ | B1 | 1 | |
| | **Total** | | **12** | |
| | **TOTAL** | | **80** | |