

General Certificate of Education
January 2005
Advanced Subsidiary Examination



**MATHEMATICS AND STATISTICS
(SPECIFICATION B)
Unit Pure 2**

MBP2

Tuesday 18 January 2005 Afternoon Session

In addition to this paper you will require:

- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 15 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBP2.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information

- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 A geometric series with positive terms is such that the fourth term is 64 times the seventh term.

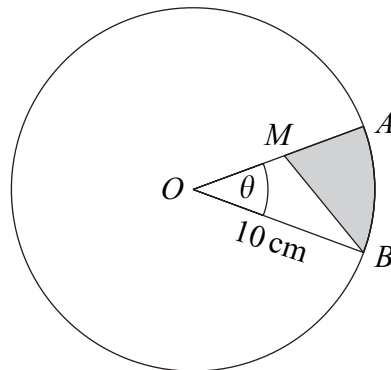
(a) (i) Prove that the common ratio, r , of the series satisfies the equation

$$r^3 = \frac{1}{64} \quad (2 \text{ marks})$$

(ii) Hence find the value of r . (1 mark)

(b) Given that the first term, a , of the series is 12, find the sum to infinity of the series. (2 marks)

2



The diagram shows a sector OAB of a circle with centre O and radius 10 cm . The angle of the sector is θ radians. The point M is the midpoint of the radius OA . The region bounded by AM , MB and the arc AB is shaded.

(a) Find, in terms of θ , the area of the shaded region. (4 marks)

(b) Given that θ is small, use the small angle approximation for $\sin \theta$ to show that the area of the shaded region is approximately $25\theta \text{ cm}^2$. (2 marks)

3 An antique is worth £ V after t years, where

$$V = 300 + 150 \ln t, \quad \text{for } t \geq 1$$

- (a) Find V when $t = 1$. (1 mark)
- (b) Find t when $V = 600$. (2 marks)
- (c) (i) Find $\frac{dV}{dt}$. (1 mark)
- (ii) Find the rate of change of V when $t = 3$. (2 marks)

4 A polynomial is given by $p(x) = 6x^3 - 7x^2 - x + 2$.

- (a) Find the value of $p\left(-\frac{1}{2}\right)$. (1 mark)
- (b) Use the factor theorem to show that $(x - 1)$ is a factor of $p(x)$. (2 marks)
- (c) Write $p(x)$ as a product of three linear factors. (3 marks)
- (d) Hence find the values of θ , in radians, in the interval $-\pi < \theta < \pi$, for which

$$6 \cos^3 \theta - 7 \cos^2 \theta - \cos \theta + 2 = 0 \quad (6 \text{ marks})$$

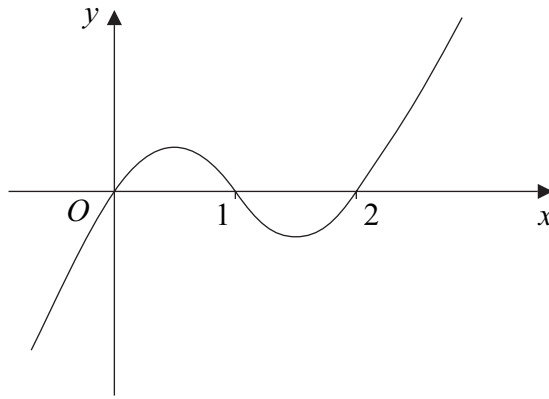
5 A curve has equation $y = e^x - 3x + 7$.

- (a) (i) Find $\frac{dy}{dx}$. (2 marks)
- (ii) The curve has a single stationary point at P . Show that the y -coordinate of P is $10 - \ln 27$. (4 marks)
- (b) (i) Find $\frac{d^2y}{dx^2}$. (1 mark)
- (ii) Hence determine whether P is a maximum or a minimum point. (2 marks)
- (c) (i) Find $\int (e^x - 3x + 7) dx$. (2 marks)
- (ii) Hence calculate the area of the region bounded by the curve $y = e^x - 3x + 7$, the x -axis and the lines $x = 0$ and $x = 2$. Give your answer in the form $e^a + b$, where a and b are integers to be found.

(You may assume that this region lies entirely above the x -axis.) (3 marks)

- 6 (a) Given that $\log_a x = m$ and $\log_a y = n$, find, in terms of m and n :
- (i) $\log_a xy$; *(1 mark)*
- (ii) $\log_a \left(\frac{x^2}{y}\right)$. *(2 marks)*
- (b) Find the value of $\log_3 6$, giving your answer to three significant figures. *(2 marks)*

- 7 The diagram shows a sketch of the curve with equation $y = x(x - 1)(x - 2)$.



- (a) Use the sketch to solve the inequality $x(x - 1)(x - 2) < 0$. *(2 marks)*
- (b) (i) Expand the brackets $x(x - 1)(x - 2)$. *(2 marks)*
- (ii) Find the x -coordinates of the two points on the curve at which the gradient of the curve is 11. *(5 marks)*
- (c) (i) Sketch the curve with equation $y = |x(x - 1)(x - 2)|$. *(2 marks)*
- (ii) The line $y = 6$ intersects the curve $y = |x(x - 1)(x - 2)|$ at the point $(3, 6)$ and at the point P . Write down the coordinates of the point P . *(1 mark)*

END OF QUESTIONS