

GCE 2004

June Series



Mark Scheme

Mathematics and Statistics B

MBP2

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

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Dr Michael Cresswell Director General

Key to Mark Scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m marks and is for	accuracy
B	mark is independent of M or m marks and is for	accuracy
E	mark is for	explanation
✓ or ft or F		follow through from previous incorrect result
cao		correct answer only
cso		correct solution only
awfw		anything which falls within
awrt		anything which rounds to
acf		any correct form
ag		answer given
sc		special case
oe		or equivalent
sf		significant figure(s)
dp		decimal place(s)
A2,1		2 or 1 (or 0) accuracy marks
-x ee		deduct x marks for each error
pi		possibly implied
sca		substantially correct approach

Abbreviations used in Marking

MC – x	deducted x marks for mis-copy
MR – x	deducted x marks for mis-read
isw	ignored subsequent working
bod	given benefit of doubt
wr	work replaced by candidate
fb	formulae book

Application of Mark Scheme

No method shown:	
Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise
More than one method / choice of solution:	
2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only
Crossed out work	do not mark unless it has not been replaced
Alternative solution using a correct or partially correct method	award method and accuracy marks as appropriate

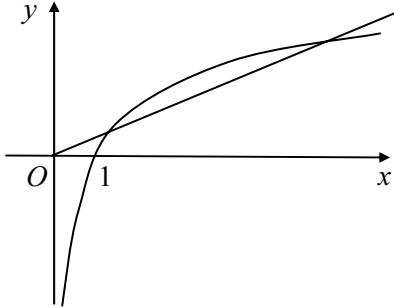
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Question number and Part	Solution	Marks	Total marks	Comments
1	$\int \frac{1}{x} dx = \ln x \dots$ $\int_2^6 \frac{1}{x} dx = \ln 6 - \ln 2$ $\dots\dots\dots = \ln 3$	B1 M1 A1	3	cao Dealing with limits correctly; F(6)–F(2) [B0M1 possible following an attempt to integrate $\frac{1}{x}$.] cso (use ISW for dec following $\ln 3$)
Total			3	
2(a)	$ar = 16; \quad ar^5 = 1$ $\Rightarrow 16r^4 = 1$ $r^4 = \frac{1}{16} \Rightarrow r = -\frac{1}{2}$ or $r = \frac{1}{2}$	B1 M1 A1	4	For either oe Elimination of a oe ag cso Full valid completion sc Clear explicit verification give maximum B2 out of 3. (accept if $-8, 4, -2$ seen)
(b)	$a = -32$ $\frac{a}{1-r} = \frac{a}{1-\left(-\frac{1}{2}\right)}$ $S_\infty = \frac{-64}{3} (= -21.3 \text{ to } 3\text{sf})$	B1 M1 A1✓	3	Accept $\frac{a}{1-r}$ quoted ft on candidate's value for a , ie $\frac{2}{3}a$ sc cand uses $r = 0.5$, gives $a = 32$ and sum to infinity = 64 (max. B0M1A1)
Total			7	
3(a)	-2 and 1	B1B1	2	
(b)	$x > 1$	B1✓	1	ft on $x >$ larger value in (a) if not $x > 1$
(c)	$x \geq 1$	B1✓		ft on $x \geq$ larger value in (a) if not $x \geq 1$
	$x = -2$	B1✓	2	ft on $x =$ smaller value in (a) if not $x = -2$
Total			5	

MBP2 (cont)

Question Number and Part	Solution	Marks	Total marks	Comments
4(a)	Angle 60° or $\frac{\pi}{3}$ radians or arc = $r\theta$	B1		Seen in part (a)
	arc : radius = $\frac{60}{360} 2\pi r : r = \pi : 3$	B1	2	ag cso
(b)	area of equilateral $\Delta = \frac{1}{2} r^2 \sin 60^\circ$	M1		$\frac{1}{2} r^2 \sin \theta$ oe stated
	$= \frac{1}{2} r^2 \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} r^2$	A1	2	ag cso
(c)	Area of sector = $\frac{1}{6} \pi r^2$	B1		
	$\frac{1}{6} \pi r^2 - \frac{\sqrt{3}}{4} r^2 = 10$	M1		Accept “area of sector – $\frac{\sqrt{3}}{4} r^2 = 10$ ”
	$r^2 = \frac{120}{2\pi - 3\sqrt{3}} = 110.3922\dots$			
	$r = 10.506\dots = 10.5$ to 3sf	A1	3	awrt 10.5
	Total		7	

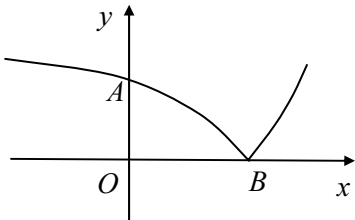
MBP2 (cont)

Question Number and Part	Solution	Marks	Total marks	Comments
5(a)(i)	$\frac{1}{4}(2) < \ln 2$ since $0.5 < 0.693..$	B1	1	Be convinced
(ii)	$\frac{1}{4}(10) > \ln 10$ since $2.5 > 2.30..$	B1	1	Be convinced
(iii)		B2,1	2	B2 graphs correct with detail. (B1 for full correct shape of $y = \ln x$; or for correct line and correct part graph of $y = \ln x$ in first quadrant from marked point (1,0))
(iv)	2 roots	B1	1	
(b)(i)	$\frac{dy}{dx} = \frac{1}{x} - \frac{1}{4}$	M1	1	One term correct
		A1	2	Accept other correct forms
(ii)	$\frac{d^2y}{dx^2} = -\frac{1}{x^2}$	A1✓	1	Only ft if $y'(x)$ has x^{-1} term
(iii)	At st. pt $y'(x) = 0 \Rightarrow \frac{1}{x} - \frac{1}{4} = 0$	M1	1	Putting their $y'(x) = 0$
	$\Rightarrow x = 4$	A1	2	
(iv)	$x^2 > 0 \Rightarrow y''(x) < 0$ {alt. $y''(4) < 0$ }	M1	1	Finding sign of 2 nd derivative, or consideration of sign of $y'(x)$ either side of st pt or relevant use of (a) oe
	\Rightarrow st pt is a maximum	B1	2	Stated
Total			12	

MBP2 (cont)

Question Number and Part	Solution	Marks	Total marks	Comments
6(a)	$(x + 2)$ is a factor of $p(x) \Rightarrow p(-2)=0$	M1		Use of $p(-2)$ or if division by $x + 2$, correctly reaches remainder $4k - 28$ { $Q(x)=6x^2 + (k - 12)x - 2k + 15$ } oe comparing coefficients
	$-48 + 4k + 18 + 2 = 0 \Rightarrow k = 7$	A1	2	ag cso
(b)	$p\left(\frac{1}{2}\right) = 6\left(\frac{1}{2}\right)^3 + k\left(\frac{1}{2}\right)^2 - 9\left(\frac{1}{2}\right) + 2$	M1		Use of $p\left(\frac{1}{2}\right)$. Accept $p\left(\frac{1}{2}\right) = 0$ stated
	$= 0 \Rightarrow (2x - 1)$ is a factor of $p(x)$	A1	2	ag Must have the conclusion
(c)	$(x + 2)(2x - 1)[3x \dots - 1]$	M1		Valid attempt at 3rd factor (coeff of x^3 or const correct)
	$p(x) \equiv (x + 2)(2x - 1)(3x - 1)$	A1	2	
(d)	$x \rightarrow \sin\theta \Rightarrow$			
	$(\sin\theta + 2)(2\sin\theta - 1)(3\sin\theta - 1) = 0$	M1		Using $x = \sin\theta$
	$\Rightarrow \sin\theta = -2; \Rightarrow$ no solution	B1✓		PI
	$\sin\theta = \frac{1}{2}; \Rightarrow \theta = \frac{\pi}{6}$	A1		Accept 0.523 or 0.524 or better
	$\theta = \frac{5\pi}{6} = 2.62$ (3 sf)	A1✓		ft on $\pi - \frac{\pi}{6}$ accept 3sf or better
	$\sin\theta = \frac{1}{3}; \Rightarrow \theta = 0.339(8..)$	A1✓		ft on cand's 3rd factor only if $ \sin\theta \leq 1$
	$\theta = 2.80(17..)$	A1✓	6	ft on $\pi - "0.339(8..)"$ Accept 2sf if 3rd sf is 0. Accept multiples of π . Ignore values outside the given interval. If answers are left in degrees deduct a maximum of 1 mark from A marks given.
	Total		12	

MBP2 (cont)

Question Number and Part	Solution	Marks	Total marks	Comments
7(a)	When $x = 0$, $y = 7$	B1	1	Accept 7 {seen at A on a sketch}
(b)	At B, $y = 0 \Rightarrow e^{3x} = 8$ $\Rightarrow x = \frac{1}{3} \ln 8 \quad (= \ln 2)$	M1 A1	2	Reaching $e^{3x} = \pm 8$, PI by $x = 0.69(31..)$ Accept any correct exact form
(c)	$\frac{dy}{dx} = -3 e^{3x}$ Gradient at B = $y'(\ln 2)$ $= -3(8) = -24$	M1 A1	2	For ke^{3x} ($k = \pm 3$ or $\pm \frac{1}{3}$). Condone any inclusion of '8' ag cso (must be exact throughout)
(d)(i)	$\int (8 - e^{3x}) dx = 8x - \frac{1}{3} e^{3x} + c$	M1 A1	2	For $8x - ke^{3x}$, $k = \frac{1}{3}$, 1 or 3 only cao {condone absence of + c}
(ii)	$\int_0^{\ln 2} (8 - e^{3x}) dx = \left[8x - \frac{1}{3} e^{3x} \right]_0^{\ln 2}$ $= \left(8 \ln 2 - \frac{1}{3} e^{3 \ln 2} \right) - \left(0 - \frac{1}{3} e^0 \right)$ $= 8 \ln 2 - \frac{8}{3} + \frac{1}{3}$ $= 8 \ln 2 - \frac{7}{3}$	M1 A1 A1	3	Dealing correctly with limits; F(b)–F(0) Condone awrt 3.21 ag cso (must be exact throughout)
(e)(i)		B1 B1	2	'Two-branch' graph covering 1st two quadrants only, with one intersection point with x-axis and left of B similar to given left portion of curve. Reasonable reflection in the x-axis of that part of the given curve that is below the x-axis.
(ii)	$e^{3x} - 8 = 19$ $x = \frac{1}{3} \ln 27 \quad (= \ln 3)$	M1 A1	2	oe Accept any exact form for this single value of x .
	Total		14	
	TOTAL		60	