

# GCE 2005

## *January Series*



# Mark Scheme

## Mathematics and Statistics B

*(MBP2)*

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Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

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*Dr Michael Cresswell Director General*

## Key to Mark Scheme

<b>M</b> .....	mark is for .....	method
<b>m</b> .....	mark is dependent on one or more M marks and is for .....	method
<b>A</b> .....	mark is dependent on M or m marks and is for .....	accuracy
<b>B</b> .....	mark is independent of M or m marks and is for .....	method and accuracy
<b>E</b> .....	mark is for .....	explanation
<b>✓ or ft or F</b> .....	follow through from previous	incorrect result
<b>CAO</b> .....	correct answer only	
<b>AWFW</b> .....	anything which falls within	
<b>AWRT</b> .....	anything which rounds to	
<b>AG</b> .....	answer given	
<b>SC</b> .....	special case	
<b>OE</b> .....	or equivalent	
<b>A2,1</b> .....	2 or 1 (or 0) accuracy marks	
<b>-x EE</b> .....	deduct $x$ marks for each error	
<b>NMS</b> .....	no method shown	
<b>PI</b> .....	possibly implied	
<b>SCA</b> .....	substantially correct approach	
<b>c</b> .....	candidate	
<b>SF</b> .....	significant figure(s)	
<b>DP</b> .....	decimal place(s)	

## Abbreviations used in Marking

<b>MC – <math>x</math></b> .....	deducted $x$ marks for mis-copy
<b>MR – <math>x</math></b> .....	deducted $x$ marks for mis-read
<b>ISW</b> .....	ignored subsequent working
<b>BOD</b> .....	given benefit of doubt
<b>WR</b> .....	work replaced by candidate
<b>FB</b> .....	formulae booklet

## Application of Mark Scheme

### **No method shown:**

Correct answer without working .....	mark as in scheme
Incorrect answer without working.....	zero marks unless specified otherwise

### **More than one method/choice of solution:**

2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only

### **Crossed out work**

do not mark unless it has not been replaced

**Alternative solution** using a correct or partially correct method

award method and accuracy marks as appropriate

**Mathematics and Statistics B Pure 2 MBP2 January 2005**

Question Number and Part	Solution	Marks	Total	Comments
1(a)(i)	$4^{\text{th}} \text{ term} = ar^3; 7^{\text{th}} \text{ term} = ar^6$ $\Rightarrow ar^3 = 64ar^6. \{ \text{Since } ar^3 \neq 0 \}$ $\Rightarrow 64r^3 = 1 \Rightarrow r^3 = \frac{1}{64}$	M1  A1	2	For use of $ar^{n-1}$ (or $ar^n$ )  <b>ag</b> Be convinced
(ii)	$r = \frac{1}{4}$	B1	1	If unsimplified look for later evidence.
(b)	$S_{\infty} = \frac{a}{1-r}$ $\Rightarrow S_{\infty} = \frac{12}{1-\frac{1}{4}} = 16$	M1  A1✓	2	ft on cand's $r$ provided $ r  < 1$
<b>Total</b>			<b>5</b>	
2(a)	$\text{Area of sector} = \frac{1}{2}r^2\theta; [= \frac{1}{2}10^2\theta]$ $\text{Area of triangle } OMB = \frac{1}{2} \times 5 \times 10 \times \sin\theta$  $\text{Area of shaded region}$ $= \frac{1}{2}10^2\theta - \frac{1}{2} \times 5 \times 10 \times \sin\theta$ $= 50\theta - 25\sin\theta$	M1  M1  m1  A1	4	Use of $\frac{1}{2}ab\sin C$ oe  Dep on at least one M
(b)	For small $\theta$ , $\sin\theta \approx \theta$ Shaded area $\approx 50\theta - 25\theta = 25\theta$	B1 B1	2	<b>ag</b> Be convinced
<b>Total</b>			<b>6</b>	
3(a)	300	B1	1	
(b)	$600 = 300 + 150 \ln t \Rightarrow \ln t = 2$ $t = e^2 \quad [= 7.38(9..)]$	M1  A1	2	As far as $\ln t = k$ Condone $t = 7.4$
(c)(i)	$\frac{dV}{dt} = \frac{150}{t}$	B1	1	
(ii)	Rate of change when $t = 3$ is $V'(3)$ $V'(3) = \frac{150}{3} = 50$	M1  A1✓	2	Recognises need for $V'(3)$ ft on $V'(t) = (p) + \frac{q}{t}, q > 0$
<b>Total</b>			<b>6</b>	

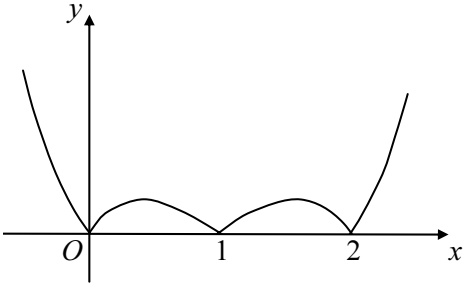
## MBP2 (cont)

Question Number and Part	Solution	Marks	Total	Comments
4(a)	$p\left(-\frac{1}{2}\right) = -\frac{6}{8} - \frac{7}{4} + \frac{1}{2} + 2 = 0$	B1	1	
(b)	$p(1) = 6 - 7 - 1 + 2$ $= 0 \Rightarrow (x - 1)$ is a factor of $p(x)$	M1 A1	2	Use of $p(1)$ . Must use F.Thm. <b>ag.</b> Must have a statement
(c)	From (a), $(2x + 1)$ is a factor $(x - 1)(2x + 1)[3x \dots - 2]$	B1 M1		Accept seen. Valid attempt at 3 <sup>rd</sup> factor (coeff of $x^3$ or const correct) or $p(2/3)$ attempted
	$p(x) \equiv (x - 1)(2x + 1)(3x - 2)$	A1	3	
(d)	$x \rightarrow \cos \theta \Rightarrow$ $(\cos \theta - 1)(2 \cos \theta + 1)(3 \cos \theta - 2) = 0$ $\Rightarrow \cos \theta = 1 \Rightarrow \theta = 0$ $\cos \theta = -\frac{1}{2}; \Rightarrow \theta = \frac{2\pi}{3} (= 2.09\dots)$ $\theta = -\frac{2\pi}{3}$ $\cos \theta = \frac{2}{3}; \Rightarrow \theta = 0.84(1\dots)$ $\theta = -0.84(1\dots)$	M1 B1 A1 A1✓ A1✓ A1✓	6	using $x = \cos \theta$ . PI  ft on “ $\frac{2\pi}{3}$ ” provided correct quadrants used for cand’s factor ft on cand’s 3 <sup>rd</sup> factor only if $ \cos \theta  \leq 1$ Deduct max of 1 from accuracy marks if answers in degrees. Ignore answers outside $-\pi < \theta < \pi$
	<b>Total</b>		<b>12</b>	

**MBP2(cont)**

Question Number and Part	Solution	Marks	Total	Comments
5(a)(i)	$\frac{dy}{dx} = e^x - 3$	B2,1,0	2	
(ii)	At st. pt $e^x - 3 = 0$ $\Rightarrow e^x = 3$ $\Rightarrow x = \ln 3 (= 1.098\dots)$ $y = 3 - 3 \ln 3 + 7 = 10 - \ln 27$	M1 A1✓ m1 A1	4	Puts $y'(x) = 0$ ft on one slip in (a) to $e^x = k$ $e^x$ to $x$ via $\ln$ <b>ag</b> Be convinced. No decimals used
(b)(i)	$\frac{d^2y}{dx^2} = e^x$	B1	1	
(ii)	$e^x > 0$ ; $y''(x) > 0$ ; $y''(\ln 3) > 0$ $\Rightarrow$ st pt is a minimum	B1✓ B1✓	2	ft on (b)(i) (any one of the three oe) ft on candidate's sign of $y''$ provided no 'dubious' statement.
(c)(i)	$\int e^x - 3x + 7 dx = e^x - \frac{3x^2}{2} + 7x \{+ c\}$	B2,1,0	2	
(ii)	Area = $\left[ e^x - \frac{3x^2}{2} + 7x \right]_0^2$ $= (e^2 - 6 + 14) - (e^0)$ $= e^2 + 7$	B1✓ M1 A1	3	PI F(2) - F(0)
<b>Total</b>			<b>14</b>	
6(a)(i)	$\log_a xy = m + n$	B1	1	
(ii)	$\log_a \left( \frac{x^2}{y} \right) = \log_a x^2 - \log_a y$ $= 2 \log_a x - \log_a y$ $= 2m - n$	M1 A1	2	Use of one law of logs. PI
(b)	$\log_3 6 = \frac{\ln 6}{\ln 3}$ oe $= 1.6309\dots = 1.63$ to 3sf	M1 A1	2	
<b>Total</b>			<b>5</b>	

## MBP2 (cont)

Question Number and Part	Solution	Marks	Total	Comments
7(a)	$x < 0; 1 < x < 2$	B1 B1	2	Deduct max of 1 from any B marks earned if $\leq$ used
(b)(i)	$x(x^2 - 3x + 2) = x^3 - 3x^2 + 2x$	B2,1,0	2	-1 each indep.error
(ii)	$y'(x) = 3x^2 - 6x + 2$ $3x^2 - 6x + 2 = 11$ $3(x^2 - 2x - 3) = 0$ $(x - 3)(x + 1) = 0$	M1 m1 A1 m1		Puts $y'(x) = 11$ Factor 3 not needed Solve or factorise. Dep on both previous M and m.
(c)(i)	$x = 3, x = -1$ 	A1	5	Need both values.
(ii)	$P(-1, 6)$	B1	1	B1 for graph correct for either $0 < x < 2$ or for both $x < 0$ and $x > 2$
	<b>Total</b>		<b>12</b>	
	<b>TOTAL</b>		<b>60</b>	