General Certificate of Education
June 2005
Advanced Subsidiary Examination

MATHEMATICS AND STATISTICS (SPECIFICATION B) **MBP1**
Unit Pure 1

Monday 23 May 2005 Morning Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 15 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBP1.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 The line AB has equation $3x + 4y = 2$.

(a) Find the gradient of AB. *(2 marks)*

(b) The point C has coordinates (5, 3) and the point B is such that BC is perpendicular to AB.

(i) Find the gradient of BC. *(2 marks)*

(ii) Show that the line BC has equation $4x - 3y = 11$. *(2 marks)*

(iii) Hence calculate the coordinates of B. *(3 marks)*

2 (a) Express $(3 + \sqrt{2})^2$ in the form $p + q\sqrt{2}$. *(2 marks)*

(b) Hence express $\dfrac{98}{(3 + \sqrt{2})^2}$ in the form $m + n\sqrt{2}$, where m and n are integers. *(3 marks)*

3 An arithmetic series has nth term u_n, where $u_n = 3n + 5$.

(a) Find the values of u_1 and u_2. *(2 marks)*

(b) State the common difference of the arithmetic series. *(1 mark)*

(c) Hence, or otherwise, find $\displaystyle\sum_{n=1}^{30} u_n$. *(3 marks)*

4 The quadratic equation $x^2 + 2kx + 2(k + 4) = 0$ has distinct real roots.

(a) Show that $k^2 - 2k - 8 > 0$. *(2 marks)*

(b) Hence find the possible values of k. *(4 marks)*

5 The function f has domain $-2 \leqslant x \leqslant 3$ and is defined by $\mathrm{f}(x) = x^2 + 1$.

(a) (i) Find the values of $\mathrm{f}(-2)$ and $\mathrm{f}(3)$. *(1 mark)*

(ii) Sketch the graph of $y = \mathrm{f}(x)$, indicating clearly the value of the intercept on the y-axis. *(3 marks)*

(iii) Hence find the range of f. *(3 marks)*

(iv) State whether the inverse of f exists, giving a reason for your answer. *(2 marks)*

(b) The function g is defined for all values of x by $\mathrm{g}(x) = (x-1)^4$.

Find $\mathrm{gf}(x)$, giving your answer in the simplest possible form. *(2 marks)*

6 (a) Prove the identity

$$\frac{3 + \sin^2\theta}{2 + \cos\theta} \equiv 2 - \cos\theta$$ *(2 marks)*

(b) Use the identity from part (a) to show that the equation

$$\frac{3 + \sin^2 2x}{2 + \cos 2x} = \frac{5}{4}$$

can be written in the form $\cos 2x = \frac{3}{4}$. *(1 mark)*

(c) Solve the equation

$$\cos 2x = \frac{3}{4}$$

in the interval $0° \leqslant x \leqslant 180°$, giving your answers to the nearest $0.1°$.

(No credit will be given for simply reading values from a graph.) *(4 marks)*

7 A curve C has equation $y = 2x + \frac{8}{x^2}$.

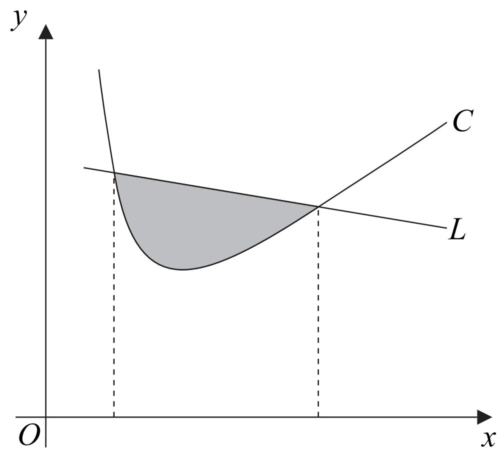
(a) (i) Find $\frac{dy}{dx}$. *(3 marks)*

(ii) Hence show that the curve has a single stationary point and find its coordinates. *(3 marks)*

(b) (i) Find $\int \left(2x + \frac{8}{x^2}\right) dx$. *(3 marks)*

(ii) Hence evaluate $\int_1^4 \left(2x + \frac{8}{x^2}\right) dx$. *(2 marks)*

(c) The curve C with equation $y = 2x + \frac{8}{x^2}$ is sketched below.



The line L with equation $x + 2y = d$ intersects the curve C at the points (1, 10) and (4, p).

(i) Find the values of the constants d and p. *(2 marks)*

(ii) Find the area of the shaded region bounded by the curve C and the line L. *(3 marks)*

END OF QUESTIONS