General Certificate of Education
January 2004
Advanced Subsidiary Examination

MATHEMATICS AND STATISTICS (SPECIFICATION B)
Unit Pure 1

MBP1

ASSESSMENT and QUALIFICATIONS ALLIANCE

Thursday 8 January 2004 Afternoon Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 15 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBP1.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 The first term of an arithmetic series is 7. The tenth term is 43.

(a) Find the common difference. *(2 marks)*

(b) Find the sum of the first fifty terms of the series. *(3 marks)*

(c) The kth term has a value greater than 1000.

(i) Show that $4k > 997$. *(2 marks)*

(ii) Find the least possible value of k. *(1 mark)*

2 The point A has coordinates $(-1, 2)$ and the line AB is parallel to the line joining the points $(7, 4)$ and $(3, 8)$.

(a) (i) Find the gradient of AB. *(1 mark)*

(ii) Show that the line AB has equation $x + y = 1$. *(1 mark)*

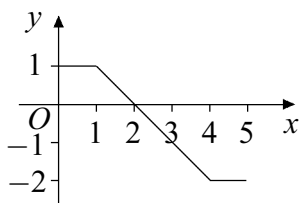
(b) The point C has coordinates $(5, 0)$ and the line BC is perpendicular to the line with equation $x + 3y = 7$.

(i) Find the gradient of BC. *(2 marks)*

(ii) Find an equation for the line BC. *(1 mark)*

(c) Find the coordinates of the point B. *(3 marks)*

3 The graph of $y = \mathrm{f}(x)$ is sketched below for $0 \leqslant x \leqslant 5$.



(a) Given that f is an even function, sketch the graph of $y = \mathrm{f}(x)$ for $-5 \leqslant x \leqslant 5$. *(2 marks)*

(b) Sketch the graph of $y = 2 - \mathrm{f}(x)$ for $0 \leqslant x \leqslant 5$. *(2 marks)*

(c) Describe fully a geometrical transformation that would map the graph of $y = \mathrm{f}(x)$ onto the graph of $y = 2 - \mathrm{f}(x)$. *(2 marks)*

4 The function g has domain $-1 \leqslant x \leqslant 2$ and is defined by $\mathrm{g}(x) = x^2 + 5$.

(a) Find $\mathrm{g}(-1)$ and $\mathrm{g}(2)$. *(2 marks)*

(b) Sketch the graph of $y = \mathrm{g}(x)$. *(2 marks)*

(c) Find the range of g. *(3 marks)*

(d) State, with a reason, whether the inverse function, g^{-1}, exists. *(2 marks)*

(e) Find $\mathrm{gg}(x)$, giving your answer in the form $x^4 + px^2 + q$. *(2 marks)*

5 The quadratic equation

$$x^2 + (3 - k)x + 5 - k^2 = 0$$

is to be considered for different values of the constant k.

(a) When $k = 7$:

(i) show that $x^2 - 4x - 44 = 0$; *(1 mark)*

(ii) find the roots of this equation, giving your answers in the form $a + b\sqrt{3}$, where a and b are integers. *(2 marks)*

(b) When the quadratic equation $x^2 + (3 - k)x + 5 - k^2 = 0$ has equal roots:

(i) show that $5k^2 - 6k - 11 = 0$; *(3 marks)*

(ii) hence find the possible values of k. *(2 marks)*

6 (a) Express the equation

$$5\sin 2x - 4\cos 2x = 0$$

in the form

$$\tan 2x = k,$$

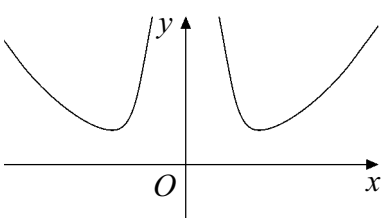
where k is a constant. *(2 marks)*

(b) Hence find all solutions of the equation

$$5\sin 2x - 4\cos 2x = 0$$

in the interval $0° < x < 180°$, giving your answers to the nearest $0.1°$. *(4 marks)*

7 A curve has equation $y = x^2 + \frac{81}{x^2}$. Its graph is sketched below.



(a) (i) Find $\frac{dy}{dx}$. *(3 marks)*

(ii) Show that the stationary points of the curve occur when $x^4 = 81$. *(2 marks)*

(iii) Hence find the x-coordinates of the stationary points. *(2 marks)*

(iv) Find the value of the y-coordinate at each stationary point. *(1 mark)*

(b) (i) Find $\int\left(x^2 + \frac{81}{x^2}\right)dx$. *(3 marks)*

(ii) Hence find the area of the region bounded by the curve, the lines $x = 1$, $x = 3$ and the x-axis. *(2 marks)*

END OF QUESTIONS