General Certificate of Education
June 2004
Advanced Level Examination

MATHEMATICS AND STATISTICS (SPECIFICATION B) Unit Mechanics 6

MBM6

Monday 21 June 2004 Morning Session

In addition to this paper you will require:
- a 12-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 15 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBM6.
- Answer **all** questions.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$ unless stated otherwise.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 A uniform solid cylinder, of radius r and mass m, is held at rest on a rough plane inclined at an angle α to the horizontal. The axis of the cylinder is perpendicular to a line of greatest slope. The cylinder is released and rolls from rest, without slipping, down the plane.

Find the acceleration of the centre of the cylinder. *(7 marks)*

2 A point P moves in a path whose polar equation is given by

$$r = \frac{4}{4 + \cos\theta}$$

with respect to a pole O and initial line OA. At any time t during the motion, $r^2\dot{\theta} = 4$.

(a) (i) Write down an expression for $r\dot{\theta}$ in terms of θ. *(2 marks)*

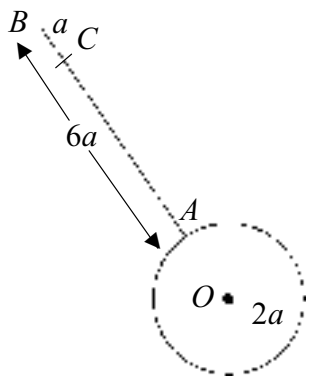
(ii) Show that $\dot{r} = \sin\theta$. *(3 marks)*

(b) Hence show that the velocity of P has magnitude

$$\sqrt{17 + 8\cos\theta}$$ *(2 marks)*

3 (a) Use integration to show that the moment of inertia of a uniform circular disc of mass m and radius a about an axis perpendicular to the disc, through the centre of the disc, is $\frac{1}{2}ma^2$. *(5 marks)*

(b) A compound pendulum consists of a uniform rod AB, of length $6a$ and mass m, together with a uniform circular disc, centre O, of mass $4m$ and radius $2a$, attached to the rod at A, as shown.



The points B, A and O lie on a straight line.

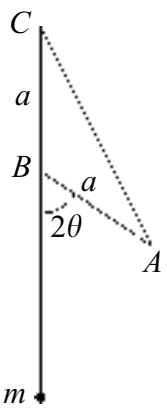
The pendulum can rotate freely in a vertical plane about a horizontal axis perpendicular to the disc and through a point C on the rod, where $AC = 5a$.

(i) Show that the moment of inertia of the compound pendulum about the axis through C is $211ma^2$. *(6 marks)*

(ii) Find the period of small oscillations of the compound pendulum. *(5 marks)*

TURN OVER FOR THE NEXT QUESTION

4 A uniform rod AB, of length a and mass $2m$, is freely pivoted at B. A light, inextensible rope, of length $4a$, attached to the rod at A, passes over a smooth peg at C, where C is at a distance a vertically above B. A particle of mass m is attached to the other end of the rope, as shown in the diagram. The angle between the rod and the vertical is 2θ, where $0 \leqslant \theta < \frac{1}{2}\pi$.



(a) Show that V, the total potential energy of the system, is given by

$$V = -mga(\cos 2\theta + 3 - 2\cos\theta)$$

taking the energy as zero at B. *(4 marks)*

(b) Show that there are two equilibrium positions for $0 \leqslant \theta < \frac{1}{2}\pi$ and determine the stability of each position. *(8 marks)*

5 Two identical uniform rods AB and BC, each of mass $3m$ and length $2a$, are rigidly joined at B so that ABC is a right angle. The body is freely hinged at A to a fixed point so that the body can move freely about A in a vertical plane.

(a) Show that AB makes an angle of $\tan^{-1}\frac{1}{3}$ with the vertical when the body is hanging in equilibrium. *(4 marks)*

(b) The body is held with AB horizontal and C above it, and is then allowed to swing.

(i) Show that the moment of inertia of the system about the horizontal axis through A is $20ma^2$. *(5 marks)*

(ii) Show that the maximum angular velocity of the body in its subsequent motion is given by $\sqrt{\dfrac{3g(1+\sqrt{10})}{10a}}$. *(5 marks)*

(iii) Calculate the vertical component of the force on the hinge as the body passes through this position of maximum angular velocity. *(4 marks)*

END OF QUESTIONS