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GCE 2004

June Series

ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark Scheme

Mathematics and Statistics B

MBM6

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Key to Mark Scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m marks and is for	accuracy
B	mark is independent of M or m marks and is for	accuracy
E	mark is for	explanation
√or ft or F		follow through from previous incorrect result
cao		correct answer only
cso		correct solution only
awfw		anything which falls within
awrt		anything which rounds to
acf		any correct form
ag		answer given
sc		special case
oe		or equivalent
sf		significant figure(s)
dp		decimal place(s)
A2,1		2 or 1 (or 0) accuracy marks
$-x$ ee		deduct x marks for each error
pi		possibly implied
sca		substantially correct approach

Abbreviations used in Marking

MC – x	deducted x marks for mis-copy
MR – x	deducted x marks for mis-read
isw	ignored subsequent working
bod	given benefit of doubt
wr	work replaced by candidate
fb	formulae book

Application of Mark Scheme

No method shown:	
Correct answer without working	**mark as in scheme**
Incorrect answer without working	**zero marks unless specified otherwise**
More than one method / choice of solution:	
2 or more complete attempts, neither/none crossed out	**mark both/all fully and award the mean mark rounded down**
1 complete and 1 partial attempt, neither crossed out	**award credit for the complete solution only**
Crossed out work	**do not mark unless it has not been replaced**
Alternative solution **using a correct or partially correct method**	**award method and accuracy marks as appropriate**

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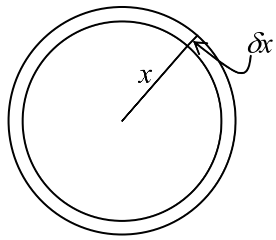
Question Number and Part	Solution	Marks	Total	Comments
1	At time t, let the cylinder have rolled a distance x down the inclined plane and have an angular velocity of ω. The speed of the centre of the cylinder is v where $v = r\omega$ Since the cylinder does not slide $v = \dot{x} = r\dot{\theta} = r\omega$	B1		
	Using forces and $G = I\ddot{\theta}$ Using $F = ma$ along the inclined plane	M1		**Alternatively** using energy The kinetic energy of the cylinder is the kinetic energy of the linear motion of the centre of mass of the cylinder plus the rotational kinetic energy of the cylinder $= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ $= \frac{1}{2}m(r\omega)^2 + \frac{1}{2}\times\frac{1}{2}mr^2\times\omega^2$
	$ma = mg\sin\alpha - F$	A1		$= \frac{3}{4}mr^2\omega^2$
	Using $G = I\ddot{\theta}$ about O, the centre of the cylinder,	M1		By conservation of energy,
	$Fr = \frac{1}{2}mr^2\ddot{\theta} = \frac{1}{2}mr^2\dot{\omega}$ $F = \frac{1}{2}mr\dot{\omega}$	A1		$mg\,x\sin\alpha = \frac{3}{4}mr^2\omega^2 = \frac{3}{4}mv^2$
	Since $v = \dot{x} = r\dot{\theta} = r\omega,\ a = r\dot{\omega}$ $ma = mg\sin\alpha - \frac{1}{2}mr\dot{\omega}$ $\frac{3}{2}ma = mg\sin\alpha$	M1		Differentiating with respect to x, $mg\sin\alpha = \frac{d}{dx}(\frac{3}{4}mv^2)$ $= \frac{d}{dv}(\frac{3}{4}mv^2)\frac{dv}{dt}\frac{dt}{dx}$ $= \frac{3}{2}mv \times a \times \frac{1}{v}$ $= \frac{3}{2}ma$
	$a = \frac{2}{3}g\sin\alpha$	A1	7	$\therefore a = \frac{2}{3}g\sin\alpha$
	Total		**7**	

MBM6 (cont)

Question Number and Part	Solution	Marks	Total	Comments
2 (a)(i)	$r\dot{\theta} = \dfrac{r^2\dot{\theta}}{r}$	M1		
	$= \dfrac{4}{\frac{4}{4+\cos\theta}}$			
	$= 4 + \cos\theta$	A1	2	
(ii)	$\dot{r} = \dfrac{\mathrm{d}}{\mathrm{d}t}\left(\dfrac{4}{4+\cos\theta}\right)$	M1		Allow for $\dfrac{\mathrm{d}}{\mathrm{d}\theta}\left(\dfrac{4}{4+\cos\theta}\right)$
	$= \dfrac{4\sin\theta}{(4+\cos\theta)^2}\dot{\theta}$	M1		
	$= \dfrac{4\sin\theta}{(4+\cos\theta)^2}\dfrac{4+\cos\theta}{\frac{4}{4+\cos\theta}}$			
	$= \sin\theta$	A1	3	
(b)	Transverse velocity, $r\dot{\theta}$, is $4+\cos\theta$			
	Radial velocity is $\sin\theta$			
	Magnitude of velocity is			
	$\sqrt{(4+\cos\theta)^2 + (\sin\theta)^2}$	M1		
	$= \sqrt{17 + 8\cos\theta}$	A1	2	
	Total		**7**	

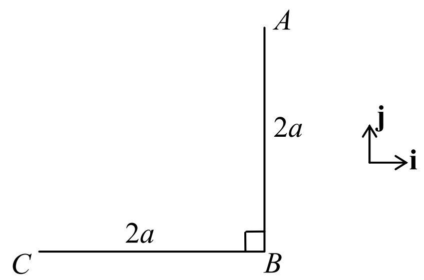
MBM6 (cont)

Question Number and Part	Solution	Marks	Total	Comments
3(a)	Consider ring shown	M1		
				
	$\delta I = 2\rho\pi x\,\delta x . x^2$			
	$= 2\pi\rho\, x^3\,\delta x$			
	$\frac{\mathrm{d}I}{\mathrm{d}x} = 2\pi\rho\, x^3$	A1		
	$I = \int_0^a 2\pi\rho x^3\,\mathrm{d}x = \pi\rho\,\frac{a^4}{2}$	M1		
	$= \frac{1}{2}ma^2$ [using $m = \pi a^2\rho$]	A1		
		B1	5	For $m = \pi a^2\rho$
(b)(i)	M of I of rod about centre is			
	$\frac{1}{3}m(3a)^2 = 3ma^2$	B1		
	M of I of rod about C is			
	$3ma^2 + m(2a)^2 = 7ma^2$	B1		
	M of I of disc about centre is			
	$\frac{1}{2}4m(2a)^2 = 8ma^2$	B1		
	M of I of disc about C is $8ma^2 + 4m(7a)^2$	M1		
	$= 204ma^2$	A1		
	M of I of compound pendulum is $211ma^2$	A1	6	
(ii)	Using $G = I\ddot{\theta}$,	M1		
	$211\,ma^2\ddot{\theta} = -\,mg.2a\sin\theta - 4mg.7a\sin\theta$	A1 A1		
	$= -\,30mga\theta$ [for small angles]			
	$\ddot{\theta} = -\frac{30g}{211a}\theta$	A1		cao
	Period is $2\pi\sqrt{\frac{211a}{30g}}$	A1√	5	ft from $\ddot{\theta}$ above
				Accept $\frac{2\pi}{\sqrt{\frac{30g}{211a}}}$
				sc 4 if correct except no '–' sign in lines 2,3,4
	Or			
	C of G 6a	(M1 A1)		
	Periodic time is $2\pi\sqrt{\frac{211a^2}{5g.6a}}$	(M1 A1)		
	$= 2\pi\sqrt{\frac{211a}{30g}}$	(A1)		
	Total		**16**	

MBM6 (cont)

Question Number and Part	Solution	Marks	Total	Comments
4 (a)	Distance of particle below B is			
	$4a - a - 2a\cos\theta$	M1		
	$= 3a - 2a\cos\theta$	A1		
	P.E. $= -mg(3a - 2a\cos\theta)$			
	$\therefore$ PE of system is			
	$-2mg\frac{a}{2}\cos 2\theta - mg(3a - 2a\cos\theta)$	M1		
	$= -mga(\cos 2\theta + 3 - 2\cos\theta)$	A1	4	**sc** 3 if energy not taken to be zero at B
(b)	$\frac{dV}{d\theta} = 2mga\sin 2\theta - 2mga\sin\theta$	M1 A1		
	$= 0 \Rightarrow$			
	$2\sin 2\theta - 2\sin\theta = 0$	M1		
	$4\sin\theta\cos\theta - 2\sin\theta = 0$			
	$\sin\theta(2\cos\theta - 1) = 0$	A1		
	$\theta = 0$ or $\frac{\pi}{3}$	A1		
	$\frac{d^2V}{d\theta^2} = 4mga\cos 2\theta - 2mga\cos\theta$	M1		
	When $\theta = 0$, this gives stable equilibrium	A1		
	When $\theta = \frac{\pi}{3}$,			
	this gives unstable equilibrium.	A1	8	
	Total		**12**	

MBM6 (cont)

Question Number and Part	Solution	Marks	Total	Comments
5(a)				
	C of G by symmetry, using **i** and **j** as shown relative to C,			
	C of G of combined body is $\frac{3}{2}a\mathbf{i} + \frac{a}{2}\mathbf{j}$	M1 A1		
	In equilibrium position, line joining A to this point is vertical			
	$\therefore \tan\phi = \frac{1}{3}$	M1 A1	4	
(b)(i)	M of I of rod about axes through A is			
	$\frac{4}{3}.3ma^2 = 4ma^2$	B1		
	M of I of rod BC is			
	$\frac{1}{3}.3ma^2 + 3m\left(\sqrt{5a^2}\right)^2 = 16ma^2$	M1A1		Condone $\frac{4}{3}.3ma^2 + 3m(2a)^2$
		B1		For $\sqrt{5}\,a$
	$\therefore$ M of I of system is $20ma^2$	A1	5	
(ii)	When AG is vertical by conservation of energy, where G is the centre of mass,			
	$\frac{1}{2}20ma^2\dot{\theta}^2 = 3mga + 6mg.\frac{\sqrt{10}}{2}a$	M1A1 M1 A1		LHS RHS 2 terms, one $3mga$ **Or** RHS Change in PE of C of G $6mg.\frac{a}{2}(1+\sqrt{10})$
	$\therefore \dot{\theta}^2 = \frac{3g(1+\sqrt{10})}{10a}$	A1	5	
(iii)	Vertical reaction at hinge is $Mg + Mh\dot{\theta}^2$	M1		Either
	$\therefore$ Reaction is $6m.g + 6m.\frac{\sqrt{10}}{2}a.\dot{\theta}^2$	M1 A1		Both
	$= 6mg + 6m.\frac{\sqrt{10}}{2}a\,\frac{3g(1+\sqrt{10})}{10a}$	A1	4	**sc** 1 for $6mg$
	$= \frac{150+9\sqrt{10}}{10}mg$ or $(15+\frac{9}{10}\sqrt{10})mg$			
	Total		**18**	
	TOTAL		**60**	