General Certificate of Education
January 2005
Advanced Level Examination

MATHEMATICS AND STATISTICS (SPECIFICATION B)
Unit Mechanics 5

MBM5

Monday 31 January 2005 Morning Session

In addition to this paper you will require:
- a 12-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 15 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBM5.
- Answer **all** questions.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$ unless stated otherwise.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 Two particles, P and Q, of masses m and $4m$ respectively, lie at rest on a smooth horizontal floor. They are joined by a light inelastic string. Initially, the string is slack.

Particle Q is set in motion with velocity $2u$ away from P.

(a) Find the speed of Q immediately after the string becomes taut. *(3 marks)*

(b) Find the impulse in the string as it becomes taut. *(2 marks)*

2 Two forces, $8\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ and $-4\mathbf{i} + 2\mathbf{j} + 14\mathbf{k}$, act at the points whose coordinates are $(2, -1, 4)$ and $(7, -3, 2)$ respectively. The three unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are mutually perpendicular.

The resultant of these forces, together with a force $\mathbf{F}$ which acts through the origin, form a couple.

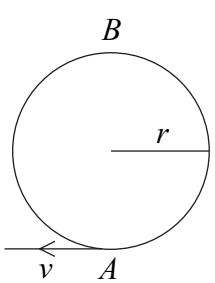
Find:

(a) (i) the force $\mathbf{F}$; *(2 marks)*

(ii) the magnitude of $\mathbf{F}$; *(2 marks)*

(b) the moment of the couple. *(5 marks)*

3 A bead, of mass m, is threaded onto a smooth circular ring, of radius r, which is fixed in a vertical plane. The bead is moving on the wire. Its speed, v, at the lowest point of its path, A, is five times its speed at the highest point, B.



(a) Show that $v = 5\sqrt{\frac{gr}{6}}$. *(4 marks)*

(b) Find the reaction of the wire on the bead, in terms of m and g, when the bead is:

(i) at B; *(3 marks)*

(ii) at a height of $\frac{1}{2}r$ above A. *(5 marks)*

4 A particle is projected down a plane inclined at an angle α to the horizontal. It is projected with velocity V at an angle θ to the inclined plane. The particle moves in a vertical plane containing the line of greatest slope.

(a) Show that the range, R, down the plane is

$$\frac{2V^2 \sin\theta \cos(\theta - \alpha)}{g \cos^2 \alpha} \qquad \textit{(6 marks)}$$

(b) Hence show that the maximum possible value of R is

$$\frac{V^2}{g \cos^2 \alpha}(1 + \sin\alpha) \qquad \textit{(4 marks)}$$

5 A hot air balloon is initially at rest on the ground. The initial mass of the balloon and its ballast is M_0 kg. A constant lift force of magnitude $M_0 g$ N is maintained using gas burners. Sand begins to leak from the balloon's ballast at a constant rate of λM_0 kg s^{-1}. As a result, the balloon begins to rise vertically.

Assume that the acceleration due to gravity is constant and you may ignore air resistance.

Assume that the velocity of the leaking sand as it leaves the balloon is the same as the velocity of the balloon at that instant.

(a) The velocity of the balloon at time t is v. Show that

$$\frac{dv}{dt} = \frac{\lambda g t}{1 - \lambda t} \qquad \textit{(5 marks)}$$

(b) Find the velocity of the balloon, in terms of g and λ, when the total mass of the balloon and its ballast has been reduced to $\frac{1}{2}M_0$ kg. *(5 marks)*

6 The motion of a particle of unit mass, moving along a straight line through O, is governed by the differential equation

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 10x = 325 \sin 3t$$

where x is the displacement from O at time t.

(a) Given that the particle starts from rest at O, solve the equation. *(11 marks)*

(b) Describe fully the nature of the motion for large values of t. *(3 marks)*

END OF QUESTIONS

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE