General Certificate of Education
January 2004
Advanced Level Examination

ASSESSMENT and QUALIFICATIONS ALLIANCE

MATHEMATICS AND STATISTICS (SPECIFICATION B)
Unit Mechanics 5

MBM5

Tuesday 27 January 2004 Afternoon Session

In addition to this paper you will require:
- a 12-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 15 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBM5.
- Answer **all** questions.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$ unless stated otherwise.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 At time t, a force, $6e^{-2t}$, acts upon a particle which is initially at rest.

Find the momentum of the particle when $t = 4$. *(4 marks)*

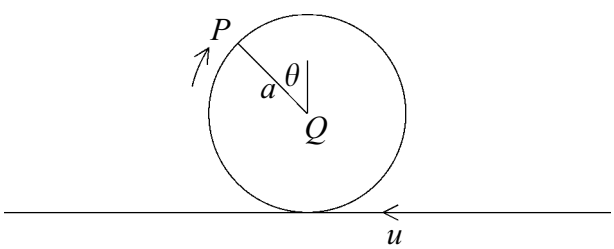
2 A force **F**, of magnitude 21 N, acts in the direction AB. The coordinates of the points A and B are $(4, -3, 5)$ and $(2, -1, 4)$ respectively. The three unit vectors **i**, **j** and **k** are in the directions of the x, y and z axes. The unit of distance is metres.

(a) Find the force **F**. *(4 marks)*

(b) Find the work done by the force **F** when it acts on a body which moves from point P $(-1, 5, 8)$ to point Q $(4, 6, -2)$. *(4 marks)*

3 In crazy golf, a golf ball is fired along a smooth track and loops the loop inside a section of track.

Model this loop as a vertical circle of radius a and centre Q, as shown in the diagram.



The golf ball is travelling at speed u as it enters the circle at the lowest point.

Model the ball as a particle P, of mass m.

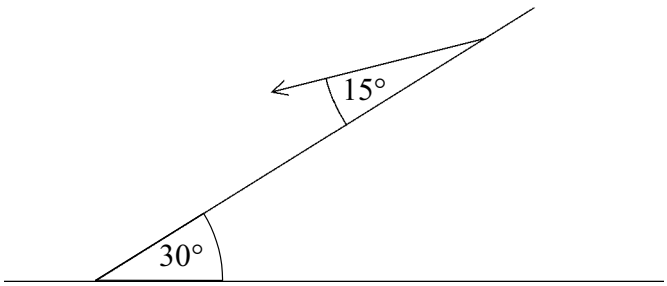
(a) Show that the reaction of the track on the particle when QP makes an angle of θ with the upward vertical is

$$\frac{mu^2}{a} - 3mg\cos\theta - 2mg$$ *(6 marks)*

(b) Given that the ball completes a vertical circle inside the track, show that

$$u \geqslant \sqrt{(5ag)}$$ *(2 marks)*

4 A particle is projected down a plane inclined at an angle 30° to the horizontal. It is projected with velocity V at an angle 15° to the inclined plane.



The particle moves in a vertical plane containing the line of greatest slope.

Show that the range down the plane is

$$\frac{2V^2}{3g}$$

(10 marks)

5 A rocket of initial mass 10 000 kg is launched from a space station where gravity can be ignored. At time t seconds after the launch, the mass of the rocket is m kg and it is travelling at $v\,\text{m s}^{-1}$. The burnt fuel is ejected at $600\,\text{m s}^{-1}$ relative to the rocket and at a constant rate of $200\,\text{kg s}^{-1}$.

(a) Write down the mass of the rocket at time t whilst the fuel is still burning. *(1 mark)*

(b) By considering linear momentum, show that

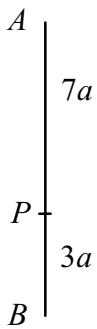
$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{600}{50 - t}$$

(7 marks)

(c) Given that the initial mass of fuel is 7000 kg, find the maximum acceleration of the rocket. *(3 marks)*

TURN OVER FOR THE NEXT QUESTION

6 A spring, of natural length $4a$ and modulus λ, has one end attached to a fixed support A, and a particle P of mass m is attached to its other end. Another spring, of natural length $2a$ and modulus $4mg$, has one end attached to P and the other end attached to a fixed support B, which is situated at a distance of $10a$ vertically below A. The system is in equilibrium in a vertical line with the upper spring stretched to a length of $7a$ and the lower spring stretched to a length of $3a$ as shown in the diagram.



(a) Show that $\lambda = 4mg$. *(4 marks)*

(b) At time $t = 0$, the particle is lowered to a distance $\frac{a}{2}$ below its equilibrium position and released from rest. The subsequent motion of P is subject to a resistance of magnitude $\frac{1}{5}mkv$, where $k^2 = \frac{6g}{a}$ and v is the speed of the particle at time t.

(i) Given that x is the downward displacement of P from its equilibrium position at time t, show that

$$10\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2k\frac{\mathrm{d}x}{\mathrm{d}t} + 5k^2x = 0$$ *(6 marks)*

(ii) Hence find x in terms of a, k and t. *(9 marks)*

END OF QUESTIONS