GCE 2005
January Series

ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark Scheme

Mathematics and Statistics B
(MBM5)

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Key to Mark Scheme

M .... mark is for .... method
m.................................... mark is dependent on one or more M marks and is for .... method
A .... mark is dependent on M or m marks and is for .... accuracy
B .... mark is independent of M or m marks and is for .... method and accuracy
E .... mark is for .... explanation
√ or ft or F ....follow through from previous incorrect result
CAO .... correct answer only
AWFW... anything which falls within
AWRT... anything which rounds to
AG .... answer given
SC .... special case
OE .... or equivalent
A2,1 .... 2 or 1 (or 0) accuracy marks
-*x* EE .... deduct *x* marks for each error
NMS .... no method shown
PI .... possibly implied
SCA .... substantially correct approach
c .... candidate
SF .... significant figure(s)
DP.. decimal place(s)

Abbreviations used in Marking

MC – *x* .... deducted *x* marks for mis-copy
MR – *x* .... deducted *x* marks for mis-read
ISW ....ignored subsequent working
BOD ....given benefit of doubt
WR ....work replaced by candidate
FB...formulae booklet

Application of Mark Scheme

No method shown:
Correct answer without working .... mark as in scheme
Incorrect answer without working.. zero marks unless specified otherwise

More than one method/choice of solution:

2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only
Crossed out work	do not mark unless it has not been replaced
Alternative solution using a correct or partially correct method	award method and accuracy marks as appropriate

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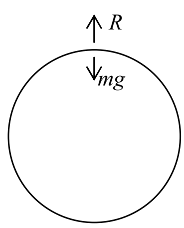
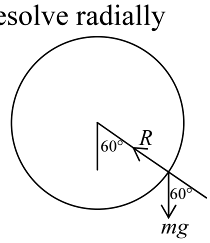
Question Number and Part	Solution	Marks	Total	Comments
1(a)	m $4m$ Initial $\rightarrow 2u$ Final $\rightarrow v$ $\rightarrow v$ C of momentum: $4m.2u = (m + 4m).\ v$ $8m.u = 5mv$ $v = \frac{8}{5}u$	 M1 A1 A1	 3	
(b)	Impulse = change in momentum Using particle *P:* Impulse $= m \times \frac{8}{5}u$ $= \frac{8}{5}mu$	 M1 A1	 2	
	Total		**5**	
2(a)(i)	$\mathbf{F} = \{(8\mathbf{i} + \mathbf{j} - 2\mathbf{k}) + (-4\mathbf{i} + 2\mathbf{j} + 14\mathbf{k})\}$ $= -4\mathbf{i} - 3\mathbf{j} - 12\mathbf{k}$	M1 A1	 2	
(ii)	Magnitude $= \sqrt{4^2 + 3^2 + 12^2}$ $= 13$	M1 A1√	 2	
(b)	Moments about origin $= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -1 & 4 \\ 8 & 1 & -2 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & -3 & 2 \\ -4 & 2 & 14 \end{vmatrix}$ $= (-2\mathbf{i} + 36\mathbf{j} + 10\mathbf{k}) + (-46\mathbf{i} - 106\mathbf{j} + 2\mathbf{k})$ $= -48\mathbf{i} - 70\mathbf{j} + 12\mathbf{k}$	M1 M1 A1 A1 A1	 5	(Use of $\mathbf{r} \times \mathbf{F}$) M1 for use of determinant or at least 2 terms correct in either moment (either moment) (second moment)
	Total		**9**	

MBM5 (cont)

Question Number and Part	Solution	Marks	Total	Comments
3(a)	Conservation of energy:			
	$\frac{1}{2}m\left(\frac{v}{5}\right)^2 + mg2r = \frac{1}{2}mv^2$	M1 A1		
	$mg4r = m\frac{24}{25}v^2$	M1		
	$v = 5\sqrt{\frac{gr}{6}}$	A1	4	
(b)(i)	At highest point speed is $\sqrt{\frac{1}{6}gr}$	B1		
				
	Consider vertical forces:			
	$R + \frac{m\left(\sqrt{\frac{1}{6}gr}\right)^2}{r} = mg$	M1		
	$R + \frac{1}{6}mg = mg$			
	$R = \frac{5}{6}mg$	A1	3	
(ii)	Using conservation of energy			**Or**
	$\frac{1}{2}mV^2 = \frac{1}{2}mv^2 - \frac{1}{2}mgr$	M1		$\frac{1}{2}mV^2 = \frac{1}{2}mv^2 - \frac{1}{2}mgr$ M1
	$= \frac{25}{12}mgr - \frac{1}{2}mgr$			$\frac{mV^2}{r} = R - mg\cos 60$ M1A1
	$= \frac{19}{12}mgr$			Eliminate V
	$V = \sqrt{\frac{19}{6}gr}$	A1		$mv^2 - mgr = Rr - mgr\cos 60$ A1
	Resolve radially			$\frac{25}{6}mg - mg = R - \frac{1}{2}mg$
				$R = \frac{11}{3}mg$ A1
	$\frac{mV^2}{r} = R - mg\cos 60$	M1 A1		M1 A0 if incorrect angle[ie not 60]
	$\frac{19}{6}mg = R - \frac{1}{2}mg$			
	$R = \frac{11}{3}mg$	A1	5	
	Total		**12**	

MBM5 (cont)

Question Number and Part	Solution	Marks	Total	Comments
4(a)	Distance perpendicular to slope: $S = V\sin\theta\, t - \frac{1}{2} g \cos\alpha\, t^2$ Strikes plane again when $s = 0$, $t = \dfrac{2v\sin\theta}{g\cos\alpha}$ [t = 0 not required]	M1 A1		
	Distance down slope: $s = V\cos\theta\, t + \frac{1}{2} g \sin\alpha\, t^2$ $= V\cos\theta \dfrac{2v\sin\theta}{g\cos\alpha} + \frac{1}{2} g \{\dfrac{2v\sin\theta}{g\cos\alpha}\}^2 \sin\alpha$ $= \dfrac{2v^2\cos\theta\sin\theta}{g\cos\alpha} + \dfrac{2v^2\sin^2\theta}{g\cos^2\alpha}\sin\alpha$ $= \dfrac{2V^2\sin\theta[\cos\theta\cos\alpha + \sin\theta\sin\alpha)}{g\cos^2\alpha}$ $= \dfrac{2V^2\sin\theta\cos(\theta-\alpha)}{g\cos^2\alpha}$	M1 A1 M1 A1	 6	
(b)	Range is $\dfrac{2V^2}{g\cos^2\alpha}\ \dfrac{1}{2}\ [\sin(2\theta-\alpha) + \sin\alpha]$ This is a maximum when $\sin(2\theta-\alpha)$ is a maximum which is 1 Hence maximum range is $\dfrac{V^2}{g\cos^2\alpha}(1+\sin\alpha)$	M1 A1 M1 A1	 4	**Or** by differentiation Max when $\theta = \dfrac{\alpha}{2} + \dfrac{\pi}{4}$ M1 A1 Substitution to give answer M1 A1
	Total		**10**	

MBM5 (cont)

Question Number and Part	Solution	Marks	Total	Comments
5(a)	Conservation of linear momentum: $(m+\delta m)(v+\delta v) - mv - \delta m.v =$ $-mg\delta t + M_0 g\delta t$	M1 A1		Needs at least 3 of the 5 terms correct
	$m\delta v = M_0 g\delta t - mg\delta t$ $m\dfrac{dv}{dt} = M_0 g - mg$			
	$m = M_0 - \lambda M_0 t$	B1		
	$(M_0 - \lambda M_0 t)\dfrac{dv}{dt} = M_0 g - (M_0 - \lambda M_0 t)g$			
	$(1 - \lambda t)\dfrac{dv}{dt} = \lambda gt$	M1		
	$\dfrac{dv}{dt} = \dfrac{\lambda gt}{1-\lambda t}$	A1	5	
(b)	$v = \int(-g + \dfrac{g}{1-\lambda t})\, dt$	M1		
	$v = -gt - \dfrac{g}{\lambda}\ln(1-\lambda t) + c$	A1		
	When $t = 0$, $v = 0$ $\therefore c = 0$ $\therefore v = -gt - \dfrac{g}{\lambda}\ln(1-\lambda t)$	A1		
	When $m = \dfrac{1}{2}M_0$ $\lambda t = \dfrac{1}{2}$ $t = \dfrac{1}{2\lambda}$	M1		
	$\therefore v = \dfrac{g}{\lambda}\ln 2 - \dfrac{g}{2\lambda}$	A1	5	
	Total		**10**	

MBM5 (cont)

Question Number and Part	Solution	Marks	Total	Comments
6(a)	Substituting $x = Ae^{nt}$ into CF $n^2 + 6n + 10 = 0$ $n = \frac{-6 \pm \sqrt{36-40}}{2}$ $= -3 \pm i$	M1		
	$\therefore$CF is $x = e^{-3t}(A\cos t + B\sin t)$	M1A1		
	PI: $x = C\sin 3t + D\cos 3t$ $\frac{dx}{dt} = 3C\cos 3t - 3D\sin 3t$ $\frac{d^2x}{dt^2} = -9C\sin 3t - 9D\cos 3t$ Substituting into $\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 10x = 325\sin 3t$	M1		M1 only for PI section if only Csin3t or Dcos3t used
	$-9C\sin 3t - 9D\cos 3t + 18C\cos 3t$ $-18D\sin 3t + 10C\sin 3t + 10D\cos 3t$ $= 325\sin 3t$	M1A1		
	(sin*t*) $C - 18D = 325$ (cos*t*) $D + 18C = 0$ $325C = 325$ $C = 1$	B1		
	$D = -18$	B1		
	$\therefore$PI is $x = \sin 3t - 18\cos 3t$ $x = e^{-3t}(A\cos t + B\sin t) + (\sin 3t - 18\cos 3t)$ When $x = 0, t = 0$ $\Rightarrow 0 = A - 18$ $\Rightarrow A = 18$	B1		
	$\frac{dx}{dt} = -3e^{-3t}(A\cos t + B\sin t) +$ $e^{-3t}(B\cos t - A\sin 3t) + 3\cos 3t + 54\sin 3t$ When $t = 0$, $\frac{dx}{dt} = 0$ $\Rightarrow 0 = -3A + B + 3$ $B = 51$	B1		
	$\therefore x = (18\cos t + 51\sin t)e^{-3t} +$ $\sin 3t - 18\cos 3t$	B1	11	Accept $x = 54e^{-3t}\cos(t - 1.232)$ $+(\sin 3t - 18\cos 3t)$ allow 54, 54.1, 54.08
(b)	When t is large, $x \approx \sin 3t - 18\cos 3t$ This is periodic	B1		
	with period $\frac{2\pi}{3}$	B1		
	and amplitude $5\sqrt{13}$	B1	3	Accept $\sqrt{325}$
	Total		**14**	
	TOTAL		**60**	