Mark scheme
January 2004

GCE

Mathematics & Statistics B

Unit MBM5

Key to mark scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m mark and is for	accuracy
B	mark is independent of M or m marks and is for	method and accuracy
E	mark is for	explanation
√ or ft or F		follow through from previous incorrect result
CAO		correct answer only
AWFW		anything which falls within
AWRT		anything which rounds to
AG		answer given
SC		special case
OE		or equivalent
A2,1		2 or 1 (or 0) accuracy marks
– *x* EE		Deduct *x* marks for each error
NMS		No method shown
PI		Perhaps implied
c		Candidate

Abbreviations used in marking

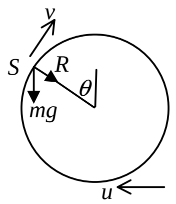
MC – *x*	deducted *x* marks for miscopy
MR – *x*	deducted *x* marks for misread
ISW	ignored subsequent working
BOD	gave benefit of doubt
WR	work replaced by candidate

Application of mark scheme

Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise

Award method and accuracy marks as appropriate to an alternative solution using a correct method or partially correct method.

Question Number and part	Solution	Marks	Total	Comments
1	Gain in momentum = $\int_0^4 F\,\mathrm{d}t$	M1		
	$= \left[-3e^{-2t}\right]_0^4$	A1 M1		M1 for limits or $+c$
	$= 3 - 3e^{-8}$	A1	4	
	Total		**4**	
2 (a)	$\overrightarrow{AB}$ = **b** – **a** = $\begin{pmatrix} -2 \\ 2 \\ -1 \end{pmatrix}$	B1		
	$\boldsymbol{F} = \lambda\begin{pmatrix} -2 \\ 2 \\ -1 \end{pmatrix}$	M1		
	Considering magnitudes $3\lambda = 21$ $\lambda = 7$	A1		
	$\boldsymbol{F} = \begin{pmatrix} -14 \\ 14 \\ -7 \end{pmatrix}$	A1	4	
(b)	Work done = **F . s**	M1		
	$= \begin{pmatrix} -14 \\ 14 \\ -7 \end{pmatrix} \bullet \begin{pmatrix} 5 \\ 1 \\ -10 \end{pmatrix}$	M1		For **s**
	= 14 joules	A1 B1	4	B1 for joules
	Total		**8**	

Question Number and part	Solution	Marks	Total	Comments
3	Let velocity at point S be v			
				
(a)	Conservation of energy			
	$\frac{1}{2}mv^2 + mga(1+\cos\theta) = \frac{1}{2}mu^2$	M1 A1		
	$v^2 = u^2 - 2ga(1+\cos\theta)$			
	$F = ma$ radially;			
	$R + mg\cos\theta = \frac{mv^2}{a}$	M1 A1		
	$R = \frac{mu^2}{a} - 2mg(1+\cos\theta) - mg\cos\theta$	M1		
	$R = \frac{mu^2}{a} - 3mg\cos\theta - 2mg$	A1	6	
(b)	For ball to complete the revolution,			
	$R \geq 0$ at top of track	M1		
	$R = \frac{mu^2}{a} - 5mg \geq 0$			
	$u \geq \sqrt{5ag}$	A1	2	
	Total		**8**	

Question Number and part	Solution	Marks	Total	Comments
4	Distance perpendicular to slope: $s = V\sin 15\, t - \frac{1}{2} g\cos 30\, t^2$ Strikes slope when $s = 0$ $t = \dfrac{2V\sin 15}{g\cos 30}$ [$t = 0$ not required] Distance along slope $s = V\cos 15\, t + \frac{1}{2} g\sin 30\, t^2$ $\therefore$ Range down slope is $V\cos 15.\dfrac{2V\sin 15}{g\cos 30} + \dfrac{1}{2} g\sin 30.\dfrac{4V^2\sin^2 15}{g^2\cos^2 30}$ $= \dfrac{2V^2\sin 15\cos 15}{g\cos 30} + \dfrac{2V^2\sin 30\sin^2 15}{g\cos^2 30}$ $= \dfrac{2V^2\sin 15}{g\cos^2 30}(\cos 30\cos 15 + \sin 30\sin 15)$ Range $= \dfrac{2V^2\sin 15\cos 15}{g\cos^2 30} = \dfrac{V^2\sin 30}{g\cos^2 30}$ $= \dfrac{2V^2}{3g}$	M1 A1 M1 A1 M1 A1 M1 A1 M1 A1	10	Or $\dfrac{4V\sin 15}{\sqrt{3}g}$ (for t) For exact answer Need to see use of sin 30 = 2 sin 15 cos 15 **sc** 9 for $\dfrac{\sqrt{3} + 4\sin^2 15}{3g} = \dfrac{2V^2}{3g}$ without justification
	Total		**10**	

Question Number and part	Solution	Marks	Total	Comments
5 (a)	$m = 10\,000 - 200t$	M1	1	Accept $10\,000 + 200t$
(b)	Initial $m \to v$ Final $m + \delta m$ $\quad -\delta m$ $\to v + \delta v$ $\quad \to v - 600$			
	Conservation of linear momentum	M1		
	$mv = (m + \delta m)(v + \delta v) - \delta m(v - 600)$	A1		
	$mv = mv + v\delta m + m\delta v - v\delta m + 600\delta m$ (to first order of δ terms) $0 = m\delta v + 600\delta m$			
	$\therefore 0 = m\frac{\mathrm{d}v}{\mathrm{d}t} + 600\frac{\mathrm{d}m}{\mathrm{d}t}$	A1		
	$\frac{\mathrm{d}m}{\mathrm{d}t} = -200$	B1		
	$\Rightarrow \therefore m\frac{\mathrm{d}v}{\mathrm{d}t} = 120\,000$	B1		
	$(10\,000 - 200t)\frac{\mathrm{d}v}{\mathrm{d}t} = 120\,000$	M1		
	$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{600}{50 - t}$	A1	7	
(c)	Maximum acceleration is when t is greatest (and fuel is still burning)	M1		
	$\therefore t = \frac{7000}{200} = 35$	B1		
	$\therefore$ Maximum acceleration is $\frac{600}{15} = 40\ \mathrm{m\,s^{-2}}$	A1	3	
	Total		**11**	

Question Number and part	Solution	Marks	Total	Comments
6 (a)	$T_{AP} = \frac{\lambda.3a}{4a} = \frac{3}{4}\lambda$			
	$T_{PB} = 4mg.\frac{a}{2a} = 2mg$	B1		Either
	Using $F = ma$ vertically $mg + T_{PB} = T_{AP}$	M1		
	$\therefore mg + 2mg = \frac{3}{4}\lambda$	A1		
	$\lambda = 4mg$	A1	4	
6 (b) (i)	When particle is moved a distance x below the equilibrium position, forces acting on it are			
	mg, $T_{AP} = \frac{\lambda.(3a+x)}{4a} = \frac{mg(3a+x)}{a}$,	M1		
	$T_{PB} = 4mg.\frac{(a-x)}{2a} = \frac{2mg}{a}(a-x)$			
	and resistance $\frac{1}{5}mk\dot{x}$	M1		All four forces
	[forces 2 and 4 are upwards] Using $F = ma$ vertically downwards			
	$m\ddot{x} = mg + T_{PB} - T_{AP} - \frac{1}{5}mk\dot{x}$	m1		Dependent on both M1 above
	$m\ddot{x} =$ $mg + \frac{2mg}{a}(a-x) - \frac{mg(3a+x)}{a} - \frac{1}{5}mk\dot{x}$	A1		
	$\ddot{x} - g - \frac{2g}{a}(a-x) + \frac{g(3a+x)}{a} + \frac{1}{5}k\dot{x} = 0$			
	$\ddot{x} + \frac{1}{5}k\dot{x} + \frac{3gx}{a} = 0$	A1		
	$10\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + 5k^2x = 0$	A1	6	

Question Number and part	Solution	Marks	Total	Comments
6 (b) (ii)	Substituting $x = Ae^{nt}$, $10n^2 + 2kn + 5k^2 = 0$			
	$n = \dfrac{-2k \pm \sqrt{4k^2 - 200k^2}}{20}$	M1		
	$= \frac{1}{10}(-k \pm 7k\text{i})$	A1		
	$x = e^{-\frac{k}{10}t}(A\cos\frac{7}{10}kt + B\sin\frac{7}{10}kt)$	M1 A1√		
	When $t = 0$, $x = \dfrac{a}{2}$, $A = \dfrac{a}{2}$	B1		
	Differentiating $\dfrac{dx}{dt} = -\dfrac{k}{10}e^{-\frac{k}{10}t}(A\cos\frac{7}{10}kt + B\sin\frac{7}{10}kt)$			
	$+ \; e^{-\frac{k}{10}t}(-\frac{7}{10}kA\sin\frac{7}{10}kt + \frac{7}{10}kB\cos\frac{7}{10}kt)$	M1 A1√		
	When $t = 0$, $\dfrac{dx}{dt} = 0$, $0 = -\dfrac{k}{10}A + \dfrac{7}{10}kB$	M1		
	$B = \dfrac{a}{14}$			
	$x = \dfrac{a}{14}e^{-\frac{k}{10}t}(7\cos\frac{7}{10}kt + \sin\frac{7}{10}kt)$	A1	9	
	Total		**19**	
	TOTAL		**60**	