General Certificate of Education
June 2005
Advanced Level Examination

MATHEMATICS AND STATISTICS (SPECIFICATION B)
Unit Mechanics 4

MBM4

Thursday 16 June 2005 Afternoon Session

In addition to this paper you will require:
- a 12-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 15 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBM4.
- Answer **all** questions.
- Take $g = 9.8 \text{ m s}^{-2}$ unless stated otherwise.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 A particle P, of mass $6m$, travelling with speed $3u$ collides directly with a particle Q, of mass $2m$, travelling in the opposite direction with speed u.

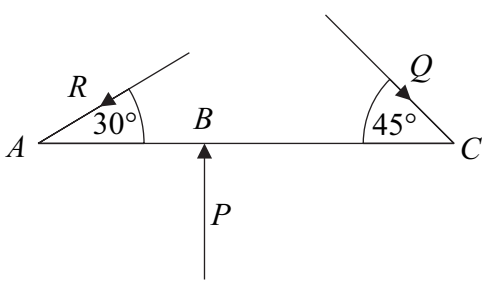
The coefficient of restitution between the two particles is $\frac{1}{4}$.

(a) Show that the speed of P after the collision is $\frac{7}{4}u$. *(6 marks)*

(b) Find the speed of Q after the collision. *(2 marks)*

2 Three horizontal forces of magnitudes P, Q and R act on a light rod, ABC, which lies on a smooth horizontal table.

The force of magnitude P acts at B, at right angles to the rod,
the force of magnitude Q acts at C, at an angle of 45° to the rod, and
the force of magnitude R acts at A, at an angle of 30° to the rod, as shown in the diagram.



The three forces together are equivalent to a couple.

Given that $P = 4\sqrt{2}$, show that $R = 4\left(\sqrt{6} - \sqrt{2}\right)$, and find Q, giving your answer in the form $a\left(b - \sqrt{c}\right)$. *(7 marks)*

3 Unit vectors **i**, **j** and **k** are defined in the east, north and vertically upwards directions respectively. The units of distance are miles, the units of time are hours and the units of velocity are miles per hour.

A car, C, is taking part in the Paris-Dakar rally and is travelling across a featureless horizontal section of the Sahara desert. A helicopter, H, is watching over the cars in the rally, ready to give assistance if required.

At noon, taken to be when $t = 0$, the car is at O, taken to be the origin, and the position vector of the helicopter is $30\mathbf{i} + 23\mathbf{j} + 0.3\mathbf{k}$.

(a) Write down the position vector of C relative to H when $t = 0$. *(1 mark)*

(b) The car has constant velocity $4\mathbf{i} - 58\mathbf{j}$, and the helicopter has constant velocity $-29\mathbf{i} - 90\mathbf{j} - 0.1\mathbf{k}$.

(i) Find the velocity of C relative to H. *(2 marks)*

(ii) Find an expression for the distance, S miles, between the helicopter and the car at time t.

You do **not** need to simplify your expression. *(4 marks)*

(iii) Find t when S^2 is a minimum.

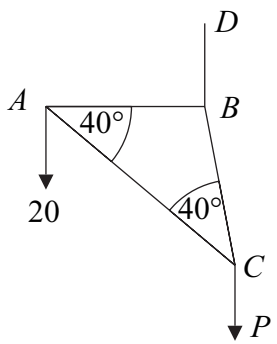
Hence state the time at which the car and the helicopter are nearest to each other. *(5 marks)*

(iv) The helicopter can be seen clearly by a person in a car when the distance between them is less than 10 miles.

Show that the helicopter can be seen clearly from the car, C, at 1 pm. *(2 marks)*

TURN OVER FOR THE NEXT QUESTION

4 The diagram shows a triangular framework ABC which is formed from three light smoothly jointed rods, AB, AC and BC.



The rods AB and BC are of equal length, and the angles at A and C are each 40°. The framework is freely suspended from a fixed point D by a light rope BD.

When a load of 20 newtons is attached at A and a load of P newtons is attached at C, the framework hangs in equilibrium in a vertical plane, with AB horizontal.

(a) Find P. *(3 marks)*

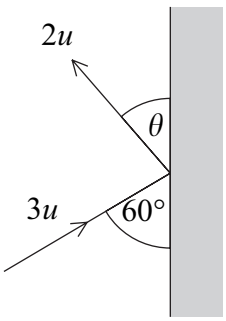
(b) Show that the magnitude of the force in the rod BC is approximately 137 newtons. *(4 marks)*

(c) Find the magnitude of the force in the rod AB. *(2 marks)*

5 A sphere of mass m, moving on a smooth horizontal surface, hits a smooth vertical wall. Just before it hits the wall, the sphere is moving at an angle of 60° to the wall with speed $3u$.

After the collision, the sphere has speed $2u$ and is moving at an angle of θ to the wall.

The diagram shows the situation viewed from above.



Model the sphere as a particle.

(a) (i) Show that $\cos\theta = \frac{3}{4}$. *(3 marks)*

(ii) Find the coefficient of restitution between the wall and the sphere. *(3 marks)*

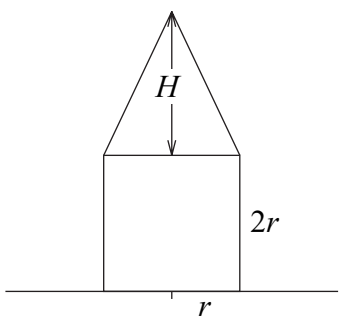
(iii) Find the angle through which the direction of the sphere is changed. *(2 marks)*

(b) Find, in terms of u and m, the impulse exerted on the sphere by the wall during this collision. *(3 marks)*

TURN OVER FOR THE NEXT QUESTION

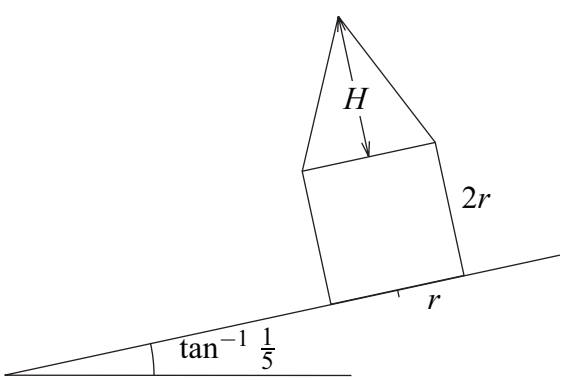
6 The diagram shows a cross-section of a child's toy, which consists of a uniform solid cylinder and a uniform solid right circular cone. One plane face of the cylinder is fixed to the plane face of the cone.

The cylinder has density ρ, radius r and height $2r$.
The cone has density 3ρ, height H and radius r.



(a) Show that the centre of mass of the toy is $\dfrac{8r^2 + 8rH + H^2}{4(2r + H)}$ from its base. *(6 marks)*

(b) The toy is now placed on a rough plane inclined at an angle of $\tan^{-1}\frac{1}{5}$ to the horizontal.



Show that the maximum value of H if the toy is not to topple is $\left(6 + 2\sqrt{17}\right)r$.
(5 marks)

END OF QUESTIONS

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