General Certificate of Education
June 2004
Advanced Level Examination

MATHEMATICS AND STATISTICS (SPECIFICATION B)
Unit Mechanics 4

MBM4

Wednesday 16 June 2004 Afternoon Session

In addition to this paper you will require:
- a 12-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 15 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBM4.
- Answer **all** questions.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$ unless stated otherwise.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 The two forces $\mathbf{i} - 5\mathbf{j} + 2\mathbf{k}$ and $2\mathbf{i} - 7\mathbf{j} - 6\mathbf{k}$, together with a third force, **F**, form a couple.

Find:

(a) the force **F**; *(3 marks)*

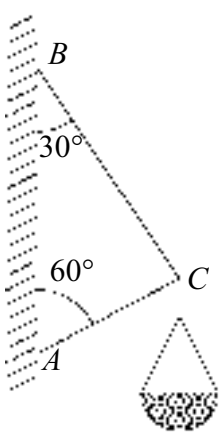
(b) the magnitude of **F**. *(2 marks)*

2 The tension, T, in a stretched elastic string is given by Hooke's Law, $T = \frac{\lambda x}{l}$, where λ is the modulus of elasticity, l is the natural length of the string and x is the extension of the string.

Find the dimensions of λ. *(4 marks)*

3 A simple framework is used to support a hanging basket of mass 10 kg.

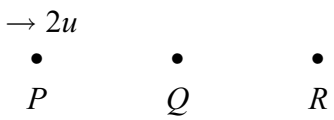
The framework comprises two light, inextensible rods, AC and BC, smoothly jointed at C. The two rods are fixed to a vertical wall at A and B, as shown in the diagram, where B is directly above A.



The basket is suspended from C.

Find the forces acting in each rod, stating whether they are in tension or compression. *(6 marks)*

4 Three particles, P, Q and R, of masses m, $3m$ and $5m$ respectively, are in a straight line on a smooth horizontal surface. Initially Q and R are at rest and P is travelling with velocity $2u$ directly towards Q. When the distance between P and Q is a, particle Q is struck a blow which results in Q travelling with the same velocity as P.



(a) Find the impulse of the blow. *(2 marks)*

(b) In the subsequent motion, Q collides directly with R. In this collision, Q is brought to rest.

(i) Find the velocity of R after the collision. *(3 marks)*

(ii) Show that the coefficient of restitution between Q and R is $\frac{3}{5}$. *(2 marks)*

(iii) Find the time after this collision when P and Q collide. Give your answer in terms of a and u. *(2 marks)*

(c) Another two particles, A and B, of masses m and M respectively, are on the smooth horizontal surface. Particle A is travelling with velocity u directly towards B, which is at rest. The two particles collide. Particle A is brought to rest by this collision.

Prove that $M \geqslant m$. *(3 marks)*

TURN OVER FOR THE NEXT QUESTION

5 The unit vectors **i**, **j** and **k** are defined respectively in the east, north and vertically upwards directions. A small airport is taken as the origin. The units of distance are kilometres and the units of velocity are kilometres per hour. Time t is measured in hours.

A small boat, B, is drifting out of control in a storm. A rescue aircraft, P, is searching for the boat and does not know its exact position.

At the start, when $t = 0$, the position vector of the boat is $11\mathbf{i} + 6\mathbf{j}$, and it is drifting with a constant velocity $3\mathbf{i} + 4\mathbf{j}$. At time $t = 0$, the position vector of the plane is $39\mathbf{i} - 7\mathbf{j} + 0.2\mathbf{k}$, and it is flying with a constant velocity $-75\mathbf{i} + 40\mathbf{j} - 0.1\mathbf{k}$.

(a) Find the position of B relative to P when $t = 0$. *(2 marks)*

(b) Find the velocity of B relative to P. *(2 marks)*

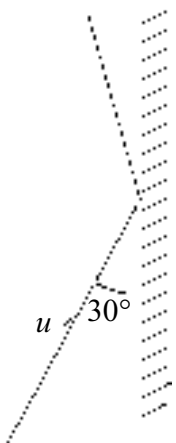
(c) Find an expression for the distance, in kilometres, which the plane and the boat are apart at time t. You do not need to simplify your expression. *(4 marks)*

(d) The range of visibility is 1 kilometre. Find the two times, in minutes after the start, when the plane is 1 kilometre from the boat. *(5 marks)*

6 A tennis ball, of mass m, is hit towards a smooth vertical wall.

Just before it hits the wall, the ball is moving horizontally at an angle of 30° to the wall with speed u. The collision changes the ball's direction of movement through 45°.

The diagram shows the situation viewed from above.



Model the ball as a particle.

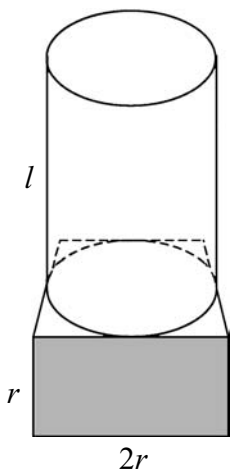
(a) Show that the coefficient of restitution between the wall and the ball is approximately 0.464. *(5 marks)*

(b) Find the percentage of the kinetic energy which has been lost during the impact with the wall. *(3 marks)*

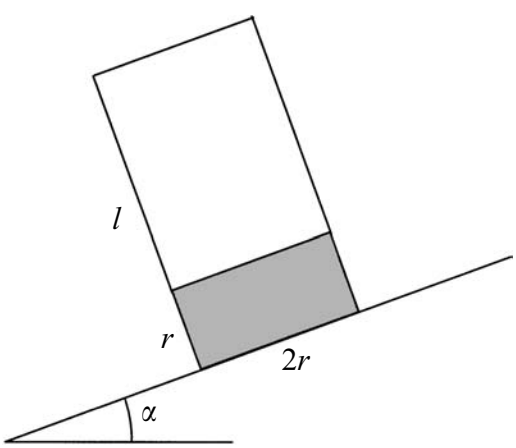
(c) Find the impulse, in terms of m and u, exerted on the ball by the wall. *(3 marks)*

TURN OVER FOR THE NEXT QUESTION

7 A trophy may be modelled as a uniform solid cylinder, of radius r and length l, fixed to the top of a cuboid of square base of side $2r$ and height r. The density of the cylinder is ρ and the density of the cuboid is 6ρ. One of the plane faces of the cylinder is fixed to one of the square faces of the cuboid so that the centres of the two faces are coincident.



The trophy is placed on a plane inclined to the horizontal at an angle α, where $\tan\alpha = \frac{1}{4}$, so that a square base of the cuboid is in contact with the inclined plane.



If the trophy is on the point of toppling, show that

$$\pi l^2 - 6\pi rl - 168r^2 = 0 \qquad \textit{(9 marks)}$$

END OF QUESTIONS

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