General Certificate of Education
January 2005
Advanced Level Examination

MATHEMATICS AND STATISTICS (SPECIFICATION B)
Unit Mechanics 4

MBM4

Friday 21 January 2005 Afternoon Session

In addition to this paper you will require:
- a 12-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 15 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBM4.
- Answer **all** questions.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$ unless stated otherwise.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 An arrow of mass m, travelling horizontally with speed $63u$, hits a block of mass $80m$, which is at rest on a smooth horizontal surface. As a result of the impact, the arrow is embedded in the block.

(a) Find, in terms of u, the speed of the combined body after the impact. *(3 marks)*

(b) Find the number of such arrows which must be fired into the block to make the block move with speed $7u$. *(4 marks)*

2 The gravitational force exerted between two spheres of masses m_1 and m_2 is

$$\frac{Gm_1m_2}{r^2}$$

where G is a constant and r is the distance between the centres of the two spheres.

(a) Find the dimensions of G. *(3 marks)*

(b) The speed, v, of a satellite orbiting a planet of mass m is given by

$$v = \frac{G^\alpha m^\beta}{r^\gamma}$$

where r is the radius of the orbit and α, β and γ are constants.

Find the values of α, β and γ for this equation to be dimensionally consistent. *(4 marks)*

3 A yacht leaves Cherbourg and sails north-west at a constant speed of $20\,\text{km}\,\text{h}^{-1}$. A customs boat is 40 km due east of Cherbourg.

The customs boat can travel at a maximum speed of $50\,\text{km}\,\text{h}^{-1}$ and intercepts the yacht in the shortest possible time.

(a) Determine the bearing on which the customs boat travels. *(5 marks)*

(b) Calculate the time taken for the customs boat to intercept the yacht. *(4 marks)*

(c) Calculate the distance that the customs boat has travelled. *(1 mark)*

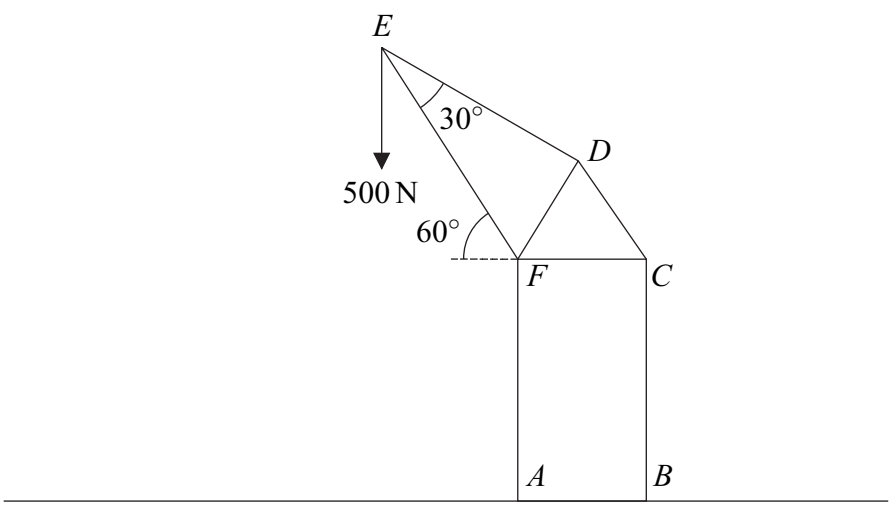
4 An engineer has designed a crane.

He modelled the crane by a framework *ABCDEF*, fixed to the ground at *A* and *B*, as shown in the diagram.

The framework is composed of light, smoothly jointed rods *AF*, *EF*, *DE*, *DF*, *DC*, *CF* and *BC*.

The framework is at rest in a vertical plane, with rod *CF* horizontal. Rods *AF* and *BC* are fixed in vertical positions. Triangle *CDF* is equilateral. Rod *EF* makes an angle of 60° with the horizontal and angle *DEF* is 30°.

A load of 500 N is attached at *E*.



(a) Find the magnitudes of the forces in the light rods *EF*, *DF* and *DE*. *(10 marks)*

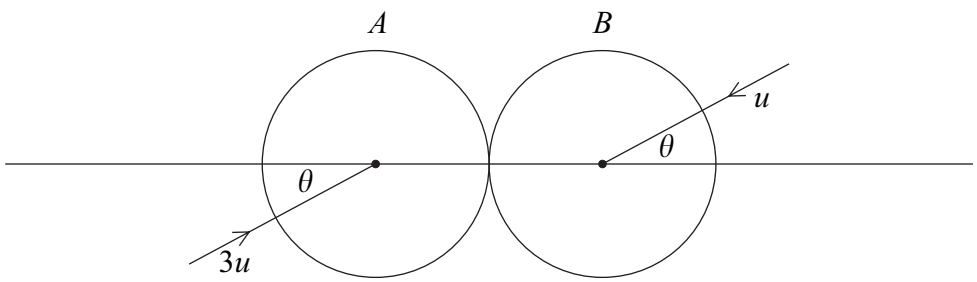
(b) State, with reasons, which of the rods *EF*, *DF* and *DE* could be replaced by ropes. *(2 marks)*

(c) State whether the magnitudes of the forces in the two rods *DC* and *CF* could be the same. Give a reason for your answer. *(2 marks)*

TURN OVER FOR THE NEXT QUESTION

5 Two smooth spheres, A and B, of equal radii and of masses m and $2m$ respectively, are moving towards each other on parallel paths in a horizontal plane. Sphere A has speed $3u$ and sphere B has speed u.

The spheres collide and the velocity of each sphere immediately before impact makes an acute angle θ with the line of centres, as shown in the diagram.

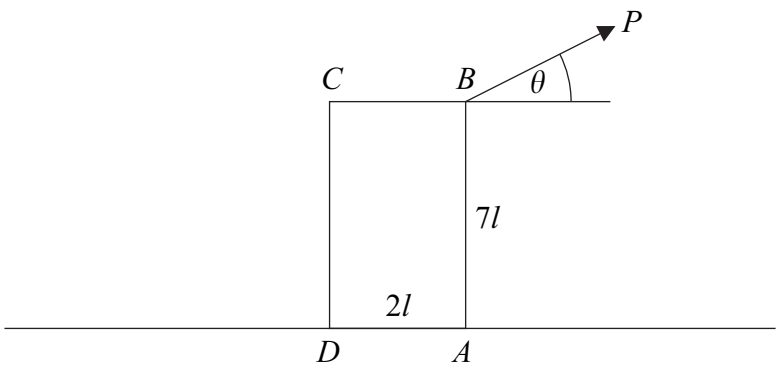


The coefficient of restitution between the spheres is e.

Find, in terms of e, u and θ, the velocity components of A and B along and perpendicular to the line of centres after the impact. *(8 marks)*

6 A uniform solid cuboid of mass M is placed on a rough horizontal floor. The cuboid has a square base of side $2l$ and a height of $7l$.

A force, P, which is gradually increasing, is applied to the midpoint of, and perpendicular to, a top edge.



This force acts as shown in the diagram, where $ABCD$ is a vertical cross-section through the centre of mass of the cuboid.

The force P makes an angle θ with the horizontal.

The coefficient of friction between the block and the rough horizontal floor is $\frac{1}{5}$.

(a) Show that, if the block is on the point of sliding,

$$P = \frac{Mg}{5\cos\theta + \sin\theta}$$ *(6 marks)*

(b) Find P when the block is on the point of toppling. *(3 marks)*

(c) Find the range of possible values of $\tan\theta$ if the block topples before it slides. *(5 marks)*

END OF QUESTIONS

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