

GCE 2004

June Series



Mark Scheme

Mathematics and Statistics B

MBM4

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

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Dr Michael Cresswell Director General

Key to Mark Scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m marks and is for	accuracy
B	mark is independent of M or m marks and is for	accuracy
E	mark is for	explanation
✓ or ft or F		follow through from previous incorrect result
cao		correct answer only
cso		correct solution only
awfw		anything which falls within
awrt		anything which rounds to
acf		any correct form
ag		answer given
sc		special case
oe		or equivalent
sf		significant figure(s)
dp		decimal place(s)
A2,1		2 or 1 (or 0) accuracy marks
-x ee		deduct x marks for each error
pi		possibly implied
sca		substantially correct approach

Abbreviations used in Marking

MC – x	deducted x marks for mis-copy
MR – x	deducted x marks for mis-read
isw	ignored subsequent working
bod	given benefit of doubt
wr	work replaced by candidate
fb	formulae book

Application of Mark Scheme

No method shown:

Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise

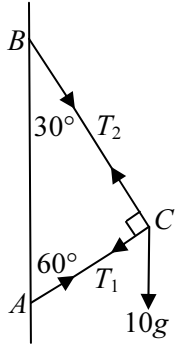
More than one method / choice of solution:

2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only

Crossed out work	do not mark unless it has not been replaced
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Alternative solution using a correct or partially correct method	award method and accuracy marks as appropriate
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Mathematics and Statistics B Mechanics 4 MBM4 June 2004

Question Number and Part	Solution	Marks	Total	Comments
1(a)	$\mathbf{F} = - \left\{ \begin{pmatrix} 1 \\ -5 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ -7 \\ -6 \end{pmatrix} \right\}$ $= \begin{pmatrix} -3 \\ 12 \\ 4 \end{pmatrix}$	M1 M1 A1	3	[- sign]
(b)	Magnitude = $\sqrt{(-3)^2 + 12^2 + 4^2}$ = 13	M1 A1✓	2	
Total			5	
2	Dimension of a force is MLT^{-2} From Hooke's law; $\lambda = \frac{IT}{x}$ $= \frac{L \times MLT^{-2}}{L}$ $= MLT^{-2}$	B1 B1 M1 A1	4	
Total			4	
3	 <p>Resolve along BC: $T_2 = 10g \cos 30$ $T_2 = 5\sqrt{3}g$ or 84.9 N</p> <p>Resolve along AC: $T_1 + 10g \sin 30 = 0$ $T_1 = -5g$ or -49 N AC is in compression. BC is in tension.</p>	M1 A1 M1 A1 B1 B1	6	Dependent on correct working Or Resolve vertically $T_1 \cos 60 + 10g = T_2 \cos 30$ M1 Horizontally $T_2 \cos 30 + T_1 \sin 60 = 0$ M1
Total			6	

MBM4 (cont)

Question Number and Part	Solution	Marks	Total	Comments
4(a)	Velocity of Q is $2u$ Impulse = $3m \cdot 2u$ = $6mu$	M1 A1	2	
b(i)	Initial $\rightarrow 2u$ $Q \ 3m \ R \ 5m$ Final $\rightarrow V$ C of momentum $3m \cdot 2u = 5mV$ $V = \frac{6}{5}u$	M1A1 A1	3	
(ii)	Restitution $e \cdot 2u = V$ $\therefore e = \frac{3}{5}$	M1 A1	2	
(iii)	Before P collides with Q again, P travels distance a at speed $2u$ \therefore Time = $\frac{a}{2u}$	M1 A1	2	
(c)	After impact velocity of B is V Conservation of momentum $mu = MV$ Restitution $eu = V$ $eMu = MV = mu$ $e \leq 1$ $m \leq M$	B1 M1 A1	3	
	Total		12	

MBM4 (cont)

Question Number and Part	Solution	Marks	Total	Comments
5(a)	$\mathbf{r}_{B \text{ rel } P} = \mathbf{r}_B - \mathbf{r}_P$ $= \begin{pmatrix} 11 \\ 6 \\ 0 \end{pmatrix} - \begin{pmatrix} 39 \\ -7 \\ 0.2 \end{pmatrix}$ $= \begin{pmatrix} -28 \\ 13 \\ -0.2 \end{pmatrix}$	M1 A1	2	sc1 for $\begin{pmatrix} 28 \\ -13 \\ 0.2 \end{pmatrix}$
(b)	$\mathbf{v}_{B \text{ rel } P} = \mathbf{v}_B - \mathbf{v}_P$ $= \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} -75 \\ 40 \\ -0.1 \end{pmatrix} = \begin{pmatrix} 78 \\ -36 \\ 0.1 \end{pmatrix}$	M1 A1	2	sc1 for $\begin{pmatrix} -78 \\ 36 \\ -0.1 \end{pmatrix}$
(c)	<p>At time t_1, $\mathbf{r}_{B \text{ rel } P} = \begin{pmatrix} -28 \\ 13 \\ -0.2 \end{pmatrix} + t \begin{pmatrix} 78 \\ -36 \\ 0.1 \end{pmatrix}$</p> $= \begin{pmatrix} -28 + 78t \\ 13 - 36t \\ -0.2 + 0.1t \end{pmatrix}$ <p>\therefore Distance apart</p> $= \sqrt{(-28 + 78t)^2 + (13 - 36t)^2 + (-0.2 + 0.1t)^2}$	M1 A1✓ M1 A1✓	4	Could be seen in (d)
(d)	<p>When distance is 1 km,</p> $(-28 + 78t)^2 + (13 - 36t)^2 + (-0.2 + 0.1t)^2 = 1$ $953.04 - 5304.04t + 7380.01t^2 = 1$ $7380.01t^2 - 5304.04t + 952.04 = 0$ $t = 0.3708.. \text{ or } 0.3478.. \text{ [hours]}$ <p>Time is 20.9.. and 22.2.. minutes</p>	M1 A1 M1 A1 A1	5	Accept 20.87
	Total		13	

MBM4 (cont)

Question Number and Part	Solution	Marks	Total	Comments
6(a)	Ball leaves wall at an angle of 15° C of momentum along the wall: $u \cos 30 = v \cos 15$ $\therefore v = \frac{u \cos 30}{\cos 15}$ Restitution: $eu \sin 30 = v \sin 15$ $\therefore e \sin 30 = \frac{\cos 30 \sin 15}{\cos 15}$ $e = \frac{\cos 30 \sin 15}{\sin 30 \cos 15}$ $= 0.464$	B1 M1 A1 M1 A1	5	
(b)	KE before impact = $\frac{1}{2} mu^2$ After impact = $\frac{1}{2} m \left(\frac{u \cos 30}{\cos 15} \right)^2$ $= 0.4019mu^2$ \therefore Percentage loss in KE is $\frac{0.098}{0.5} \times 100$ $= 19.6\%$	B1 M1 A1	3	Dependent on B1 Accept 19.5 or 19.7
(c)	Impulse = change in momentum perpendicular to the wall $= mu \sin 30 + mv \sin 15$ $= mu(0.5 + 0.232)$ $= 0.732mu$ or $\frac{1}{2}(1 + e)mu$	M1 m1 A1	3	+ Accept $mu(\sin 30 + \cos 30 \tan 15)$
	Total		11	

MBM4 (cont)

Question Number and Part	Solution	Marks	Total	Comments
7	<p>Mass of cylinder is $\pi(r)^2 l \cdot \rho = \pi r^2 l \rho$</p> <p>Mass of cuboid is $(2r)^2 \cdot r \cdot 6\rho = 24r^3 \rho$</p> <p>Taking moments about A (at point of toppling)</p> $24r^3 \rho \left(\frac{r}{2} \sin \alpha - r \cos \alpha \right) + \pi r^2 l \rho \left\{ \left(\frac{l}{2} + r \right) \sin \alpha - r \cos \alpha \right\} = 0$ $12r^2 \tan \alpha - 24r^2 + \pi \frac{l^2}{2} \tan \alpha \dots$ $\dots + \pi l \tan \alpha - \pi l = 0$ $\pi l^2 - 6\pi l - 168r^2 = 0$ <p>Or using centre of gravity</p> <p>Mass of cylinder is $\pi(r)^2 l \cdot \rho = \pi r^2 l \rho$</p> <p>Mass of cuboid is $(2r)^2 \cdot r \cdot 6\rho = 24r^3 \rho$</p> <p>Let \bar{x} be distance of C of G from base of trophy</p> <p>Taking moments about plane of base</p> $24r^3 \rho \frac{r}{2} + \pi r^2 l \rho \left(\frac{l}{2} + r \right) = \{24r^3 \rho + \pi r^2 l \rho\} \bar{x}$ $12r^2 + \frac{\pi}{2} l^2 + \pi l = \{24r + \pi l\} \bar{x}$ $\bar{x} = \frac{12r^2 + \frac{\pi}{2} l^2 + \pi l}{24r + \pi l}$ <p>If, on point of toppling, C of G is vertically above A</p> $\bar{x} \sin \alpha = r \cos \alpha$ $(24r^2 + 2\pi l + \pi l^2) \tan \alpha = 48r^2 + 2\pi l$ $6r^2 + \frac{1}{2} \pi l + \frac{1}{4} \pi l^2 = 48r^2 + 2\pi l$ $\pi l^2 - 6\pi l - 168r^2 = 0$	<p>B1</p> <p>B1</p> <p>M1</p> <p>M1 A1</p> <p>M1A1</p> <p>M1</p> <p>A1</p> <p>(B1)</p> <p>(B1)</p> <p>(M1)</p> <p>(A1)</p> <p>(M1)</p> <p>(A1)</p> <p>(M1)</p> <p>(M1)</p> <p>(A1)</p>	9	Condone not using ρ as long as 6 used
	Total		9	
	TOTAL		60	