ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark scheme
January 2004

GCE

Mathematics & Statistics B

Unit MBM4

Key to mark scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m mark and is for	accuracy
B	mark is independent of M or m marks and is for	method and accuracy
E	mark is for	explanation
√ or ft or F		follow through from previous incorrect result
CAO		correct answer only
AWFW		anything which falls within
AWRT		anything which rounds to
AG		answer given
SC		special case
OE		or equivalent
A2,1		2 or 1 (or 0) accuracy marks
– *x* EE		Deduct *x* marks for each error
NMS		No method shown
PI		Perhaps implied
c		Candidate

Abbreviations used in marking

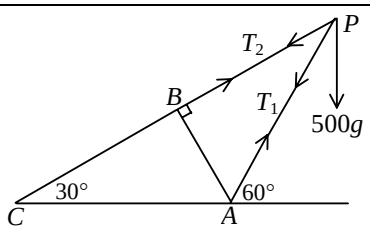
MC – *x*	deducted *x* marks for miscopy
MR – *x*	deducted *x* marks for misread
ISW	ignored subsequent working
BOD	gave benefit of doubt
WR	work replaced by candidate

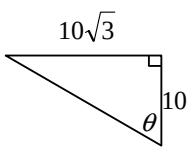
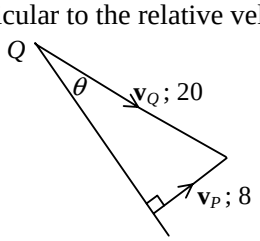
Application of mark scheme

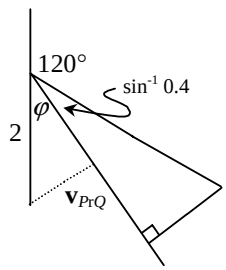
Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise

Award method and accuracy marks as appropriate to an alternative solution using a correct method or partially correct method.

Question number and part	Solution	Marks	Total marks	Comments
1(a)	Change in momentum is			
	$0.04 \times 12 - 0.04 \times -8$	B1 M1		Conversion to kg
		B1		Correct signs
	Impulse is 0.8 Ns	A1	4	– 0.8 B2 M1
(b)	Using Force × time = impulse			
	Force = $\frac{0.8}{0.05}$	M1		
	= 16 N	A1√	2	ft
	Total		**6**	
2	Dimensions of a and g are LT^{-2}	B1		
	Dimension of v is LT^{-1}	B1		
	$\lambda = \frac{LT^{-2}}{(LT^{-1})^2}$	M1		
	$= L^{-1}$	A1	4	
	Total		**4**	
3(a)	$\mathbf{F} = (7\mathbf{i} + 2\mathbf{j}) + (-3\mathbf{i} + 4\mathbf{j}) + (\mathbf{i} + 6\mathbf{j})$	M1		
	$= 5\mathbf{i} + 12\mathbf{j}$	A1		
	Magnitude of $\mathbf{F}$ is $\sqrt{5^2 + 12^2}$	M1		
	= 13	A1	4	
(b)	Moments about O;			Can take moments about $(x, 0)$ etc
	$12x$	M1 A1		
	$= 3\times1 + 4\times4 + 6\times8 - 1\times2 + 2\times3 + 7\times5$	M1 A1		
	$12x = 106$			
	$x = \frac{106}{12} = \frac{53}{6}$	A1	5	
	Point is $(\frac{53}{6}, 0)$			Can use printed result
	Total		**9**	

Question number and part	Solution	Marks	Total marks	Comments
4(a)	Using conservation of momentum	M1		
	$3m\begin{pmatrix}7\\-8\end{pmatrix} + m\begin{pmatrix}2\\5\end{pmatrix} = m\begin{pmatrix}5\\-4\end{pmatrix} + 3m\,\mathbf{v}$	A1		
	$\begin{pmatrix}21\\-24\end{pmatrix} + \begin{pmatrix}2\\5\end{pmatrix} = \begin{pmatrix}5\\-4\end{pmatrix} + 3\,\mathbf{v}$	M1		
	$3\,\mathbf{v} = \begin{pmatrix}18\\-15\end{pmatrix}$			
	$\mathbf{v} = \begin{pmatrix}6\\-5\end{pmatrix}$	A1	4	
(b)	Change in momentum =			
	$m\begin{pmatrix}5\\-4\end{pmatrix} - m\begin{pmatrix}2\\5\end{pmatrix}$	M1		M1 for $-3m\mathbf{i} + 9m\mathbf{j}$
	$= 3m\mathbf{i} - 9m\mathbf{j}$	A1	2	**sc** 1 for $3\mathbf{i} - 9\mathbf{j}$
(c)	Direction is $\mathbf{i} - 3\mathbf{j}$ oe	B1√		ft from (b)
	Line of centres is parallel to the change in momentum	B1	2	
	Total		**8**	
5(a)				
	Resolve horizontally at P			
	$T_1 \cos 60 + T_2 \cos 30 = 0$	M1 A1		
	$T_1 + \sqrt{3}\,T_2 = 0$			
	Resolve vertically at P			
	$T_1 \sin 60 + T_2 \sin 30 = -500\,g$	M1 A1		
	$T_1\sqrt{3} + T_2 = -1000\,g$			
	$T_1 = -500\sqrt{3}\,g$; $T_2 = 500g$			
	Force in AP is $500\sqrt{3}\,g$ N or $4900\sqrt{3}$ N or 8490 N in compression	A1		**sc** 5 if g omitted
	Force in BP is $500g$ N or 4900 N in tension	A1	6	
(b)	Force in AB is zero since forces at B are in equilibrium and the other two forces in BC and BP are parallel.	B1 B1	2	
	Total		**8**	

Question number and part	Solution	Marks	Total marks	Comments
6(a)	Moments about A			
	$P\ 4l\cos\alpha$	M1 A1		M1 awarded for moments about A even when on horizontal floor or if $P.4l$ seen
	$= mg(\ l\cos\alpha - 2l\sin\alpha\ )$	A1		
	$P = \dfrac{\cos\alpha - 2\sin\alpha}{4\cos\alpha}mg$	A1	4	m instead of mg used penalise one A1 in question
(b)	Resolve along the plane			
	$F - P\cos\alpha = mg\sin\alpha$	M1 A1		
	Resolve perpendicular to the plane			
	$P\sin\alpha + R = mg\cos\alpha$	M1 A1		
	Using $F = \mu R$	B1		
	$mg\sin\alpha + P\cos\alpha = \mu(mg\cos\alpha - P\sin\alpha)$			
	$P\cos\alpha + \mu P\sin\alpha = \mu mg\cos\alpha - mg\sin\alpha$	M1		
	$P = \dfrac{\mu\cos\alpha - \sin\alpha}{\cos\alpha + \mu\sin\alpha}mg$	A1	7	Accept $P = \dfrac{\mu - \tan\alpha}{1 + \mu\tan\alpha}mg$
	Total		**11**	
7(a)	Speed of Q is 20 km/h	B1		
				
	$\tan\theta = \dfrac{10\sqrt{3}}{10}$	M1		
	Bearing is 120°	A1	3	
b(i)	Ship P will travel so that v_P is perpendicular to the relative velocity	M1		(If not gained, can gain M1 in (ii) and all marks in (iii))
				
	$\sin\theta = \dfrac{8}{20} = 0.4$	m1		Dependent on M1 above
	$\theta = 23.6°$	A1		
	Bearing of ship P is 054°	B1	4	Dependent on first M1 Accept 053.6°
(ii)	Velocity of P is $8\sin 53.6\mathbf{i} + 8\cos 53.6\mathbf{j}$	B1		Dependent on M1,M1 in (i)
	Velocity of Q relative to P is $v_Q - v_P$			
	$= (10\sqrt{3}\,\mathbf{i} - 10\mathbf{j}) - (6.439\mathbf{i} + 4.7498\mathbf{j})$	M1		
	$= 10.88\mathbf{i} - 14.75\mathbf{j}$			
	$= 11\mathbf{i} - 15\mathbf{j}$ [to 2 significant figures]	A1	3	

Question number and part	Solution	Marks	Total marks	Comments
7(b)(iii)				
	$\varphi = 90 - (30 + \sin^{-1} 0.4)$	M1		
	$= 36.42$	A1		
	Minimum distance is $2 \sin 36.42$	M1		
	$= 1.187$ km	A1	4	
	OR			
(iii)	Position of Q relative to P is			
	$2\mathbf{j} + (10.88\mathbf{i} - 14.75\mathbf{j})\, t$			
	Distance apart, D, is			
	$\sqrt{(10.88t)^2 + (2-14.75t)^2}$	(M1)		
	$D^2 = 334.486t^2 - 58.8t + 4$	(A1)		
	$\frac{dD^2}{dt} = 668.972t - 58.8$	(M1)		
	$= 0$ when min distance,			
	when $t = \frac{58.8}{668.972} = 0.0879$			
	Minimum distance is $\sqrt{1.4094}$			
	$= 1.187$ km	(A1)	(4)	
	OR			
	Accept from printed result			
(iii)	Position of Q relative to P is			
	$2\mathbf{j} + (11\mathbf{i} - 15\mathbf{j})\, t$			
	Distance apart, D, is $\sqrt{(11t)^2 + (2-15t)^2}$	(M1)		
	$D^2 = 346t^2 - 60\, t + 4$	(A1)		
	$\frac{dD^2}{dt} = 692t - 60$	(M1)		
	$= 0$ when min distance,			
	when $t = \frac{60}{692} = 0.0867$			
	Minimum distance is $\sqrt{1.442948}$			
	$= 1.201$ km	(A1)	(4)	
	Total		**14**	
	TOTAL		**60**	