

General Certificate of Education  
January 2005  
Advanced Level Examination



**MATHEMATICS AND STATISTICS  
(SPECIFICATION B)  
Unit Mechanics 3**

**MBM3**

Tuesday 25 January 2005 Morning Session

**In addition to this paper you will require:**

- a 12-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 45 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBM3.
- Answer **all** questions.
- Take  $g = 9.8 \text{ m s}^{-2}$  unless stated otherwise.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

**Information**

- The maximum mark for this paper is 80.
- Mark allocations are shown in brackets.

**Advice**

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

---

Answer **all** questions.

---

**1** A car is travelling along a straight horizontal road. When the car is at a distance of 48 metres from a set of traffic lights, it is travelling at  $20 \text{ m s}^{-1}$ . The car slows down uniformly so that it has a speed of  $4 \text{ m s}^{-1}$  as it passes the traffic lights.

(a) Show that the acceleration of the car is  $-4 \text{ m s}^{-2}$ . *(2 marks)*

(b) How far past the traffic lights does the car travel before it stops? *(3 marks)*

(c) Calculate the total time that it takes the car to stop. *(2 marks)*

(d) The car has mass 1100 kg. As the car slows down, it experiences a resistance force that has a constant magnitude of 200 newtons, and a force due to the brakes of magnitude  $F$  newtons.

Find  $F$ . *(3 marks)*

**2** A particle, of mass 2 kg, is set into motion up a rough slope inclined at  $40^\circ$  to the horizontal. The coefficient of friction between the particle and the slope is 0.3. Assume that there is no air resistance acting on the particle.

(a) Find the magnitude of the friction force that acts on the particle. *(3 marks)*

(b) Find the acceleration of the particle as it moves up the slope. *(3 marks)*

**3** A particle moves so that, at time  $t$  seconds, its position vector,  $\mathbf{r}$  metres, is given by

$$\mathbf{r} = 4e^{-t}\mathbf{i} + (6t + 3e^{-t})\mathbf{j}$$

The unit vectors  $\mathbf{i}$  and  $\mathbf{j}$  are perpendicular.

(a) Show that the velocity of the particle is  $(-4\mathbf{i} + 3\mathbf{j}) \text{ m s}^{-1}$  when  $t = 0$ . *(3 marks)*

(b) Find an expression for the acceleration of the particle at time  $t$ . *(2 marks)*

(c) Find the magnitude of the acceleration of the particle when  $t = 0$ . *(2 marks)*

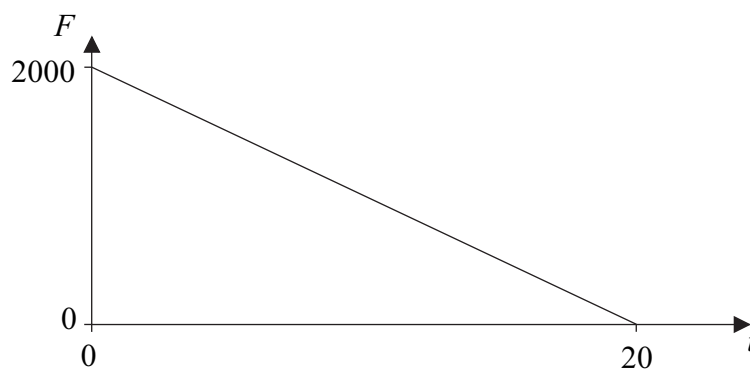
(d) Describe what happens to the velocity of the particle as  $t$  becomes large. *(2 marks)*

- 4 A small sphere, of mass 5 kg, is attached to one end of a light elastic string. The other end of the string is fixed to a smooth horizontal surface at the point  $O$ . The string has modulus of elasticity 40 N and natural length 2 metres.

The sphere is released from rest at the point  $P$ , which is on the horizontal surface 5 metres from  $O$ .

- (a) Show that the elastic potential energy of the string is 90 J when the sphere is released. (2 marks)
- (b) Show that the speed of the sphere is  $6 \text{ m s}^{-1}$  when the string becomes slack. (3 marks)
- (c) Find the speed of the sphere when it is 3 metres from  $O$ . (5 marks)

- 5 A car, of mass 1000 kg, is travelling along a straight horizontal road. Its brakes are applied for a period of 20 seconds. The graph below shows how the magnitude of the braking force,  $F$  newtons, varies with time,  $t$  seconds, for  $0 \leq t \leq 20$ .



- (a) The magnitude of the braking force, at time  $t$ , can be expressed as

$$F = P - Qt$$

where  $P$  and  $Q$  are constants.

Use the graph to find the values of  $P$  and  $Q$ . (2 marks)

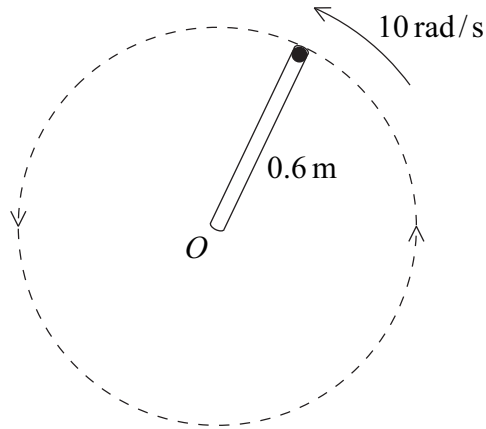
- (b) The braking force is the only horizontal force that acts on the car. Show that the acceleration of the car, at time  $t$ , is given by

$$a = \frac{t}{10} - 2 \quad (2 \text{ marks})$$

- (c) The car comes to rest when  $t = 20$ . Find an expression for the velocity of the car at time  $t$ . (4 marks)
- (d) Find the distance that the car travels in the 20 seconds. (5 marks)

- 6 The mass of a car is 1500 kg. The car travels up a slope inclined at  $5^\circ$  to the horizontal. When the car moves at a speed of  $v \text{ m s}^{-1}$ , it is subject to a resistance force of magnitude  $30v$  newtons. Model the car as a particle.
- (a) Draw a diagram to show the forces acting on the car. *(1 mark)*
- (b) Show that the power output of the car when it travels up the slope at a constant speed of  $10 \text{ m s}^{-1}$  is 15 800 W, correct to 3 significant figures. *(4 marks)*
- (c) The car has a maximum power output of 35 000 W. Find the maximum possible speed of the car as it travels, from rest, up the slope. *(6 marks)*
- 7 A particle moves with constant acceleration  $(4\mathbf{i} + 2\mathbf{j}) \text{ m s}^{-2}$ . At time  $t = 0$ , the particle is at the origin and has velocity  $(3\mathbf{i} - 10\mathbf{j}) \text{ m s}^{-1}$ . The unit vectors  $\mathbf{i}$  and  $\mathbf{j}$  are directed east and north respectively.
- (a) Show that the velocity of the particle at time  $t$  seconds is
- $$(3 + 4t)\mathbf{i} + (2t - 10)\mathbf{j} \quad \text{(*2 marks*)}$$
- (b) Find the time when the particle is travelling due east. *(2 marks)*
- (c) (i) Find the position vector of the particle when  $t = 10$ . *(3 marks)*
- (ii) Find the velocity of the particle when  $t = 10$ . *(1 mark)*
- (d) The particle stops accelerating when  $t = 10$  and then moves with constant velocity. Find the distance of the particle from the origin after it has been travelling with this constant velocity for a further 10 seconds. *(6 marks)*

- 8 The diagram shows a smooth tube, sealed at each end, which rotates about one end. There is a small sphere inside the tube. The tube rotates in a vertical plane, so that the sphere remains at the end of the tube and describes a vertical circle of radius 0.6 metres. The mass of the sphere is 0.05 kg and the tube rotates with a constant angular speed of 10 radians per second.



- (a) Calculate the magnitude of the acceleration of the sphere. *(2 marks)*
- (b) Find the magnitude of the horizontal reaction force exerted by the end of the tube on the sphere when the tube is horizontal. *(2 marks)*
- (c) Find the magnitude of the vertical reaction force exerted by the end of the tube on the sphere when the sphere is at its lowest point. *(3 marks)*

**END OF QUESTIONS**

**THERE ARE NO QUESTIONS PRINTED ON THIS PAGE**

**THERE ARE NO QUESTIONS PRINTED ON THIS PAGE**

**THERE ARE NO QUESTIONS PRINTED ON THIS PAGE**