

General Certificate of Education
January 2004
Advanced Level Examination



**MATHEMATICS AND STATISTICS
(SPECIFICATION B)
Unit Mechanics 3**

MBM3

Friday 23 January 2004 Morning Session

In addition to this paper you will require:

- a 12-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 45 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBM3.
- Answer **all** questions.
- Take $g = 9.8 \text{ m s}^{-2}$ unless stated otherwise.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information

- The maximum mark for this paper is 80.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 As a lift moves upwards its motion has three stages.

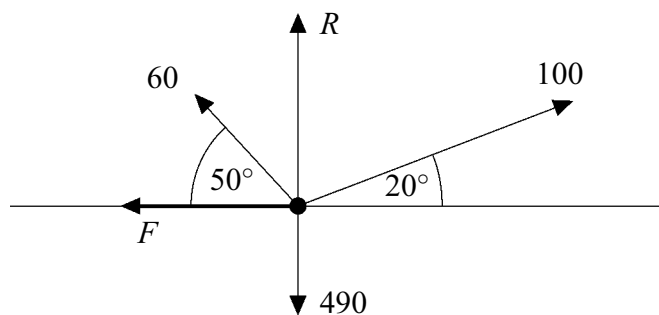
Stage I It accelerates uniformly, from rest, at 0.2 m s^{-2} , for 8 seconds.

Stage II It then travels at a constant speed for 3 seconds.

Stage III It then decelerates uniformly, stopping when it has travelled a further 1 metre.

- (a) (i) Find the distance travelled by the lift in stage I. (2 marks)
- (ii) Find the speed of the lift at the end of stage I. (2 marks)
- (iii) Find the total distance travelled by the lift. (2 marks)
- (b) Find the total time for which the lift is moving. (3 marks)
- (c) The lift and its passengers have a total mass of 600 kg. The lift is supported by a single cable. Find the tension in the cable when the lift is accelerating upwards at 0.2 m s^{-2} . (3 marks)

2 A box, of mass 50 kg, moves in a straight line on a rough horizontal surface. The diagram below shows **all** the forces acting on the box, as it moves. The magnitude of each force is in newtons. The box is modelled as a particle.



Note that the weight of the box has been included in the diagram.

- (a) Find R , the magnitude of the normal reaction force acting on the box. (3 marks)
- (b) The box accelerates at 0.5 m s^{-2} .
- (i) Show that F , the magnitude of the friction force acting on the box, is approximately 30.4. (4 marks)
- (ii) Find the coefficient of friction between the box and the surface. (2 marks)

- 3 A diver has mass 65 kg. She dives from a fixed diving board, which is 6 metres above the level of the water in the pool. When the diver leaves the board, she is travelling vertically upwards and has speed 2 m s^{-1} .

Model the diver as a particle. Assume that there are no resistance forces acting on the diver as she moves through the air and that she does not hit the board on the way down.

- (a) (i) Calculate the kinetic energy of the diver when she leaves the board. (2 marks)
- (ii) By using an energy method, calculate the maximum height of the diver above the diving board. (2 marks)
- (b) (i) Find the kinetic energy of the diver when she hits the water. (3 marks)
- (ii) Hence calculate the speed of the diver when she hits the water. (2 marks)

- 4 A particle moves on a straight line. At time t seconds its acceleration, $a \text{ m s}^{-2}$, is given by

$$a = 20 \sin 4t$$

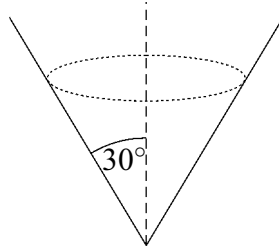
- (a) Initially the particle is at rest. Find an expression for the velocity of the particle at time t . (4 marks)
- (b) Initially the displacement of the particle from the origin is 0.8 metres. Find an expression for the displacement of the particle at time t . (4 marks)

- 5 A particle moves in a horizontal plane. The unit vectors \mathbf{i} and \mathbf{j} are perpendicular and lie in the horizontal plane.

Initially the particle is at the origin and has velocity $(5\mathbf{i} - 2\mathbf{j}) \text{ m s}^{-1}$. It moves with constant acceleration. At time $t = 10$ it has position vector $(40\mathbf{i} - 15\mathbf{j})$ metres.

- (a) Show that the velocity of the particle is $(3\mathbf{i} - \mathbf{j}) \text{ m s}^{-1}$ when $t = 10$. (4 marks)
- (b) Find the acceleration of the particle. (3 marks)
- (c) The mass of the particle is 15 kg. Find the magnitude of the resultant force acting on the particle. (4 marks)

- 6 A particle, of mass 3 kg, describes a horizontal circular path on the inside surface of a smooth cone, as shown in the diagram.



The radius of the circle is 0.5 metres and the semi-vertical angle of the cone is 30° . The particle moves at a constant speed.

- (a) (i) Show that the magnitude of the normal reaction force on the particle is 58.8 N. (3 marks)
- (ii) Find the speed of the particle. (4 marks)
- (b) The particle moves on the same cone, but in a horizontal circle of greater radius than before.
- (i) What happens to the magnitude of the normal reaction force? (1 mark)
- (ii) What happens to the speed of the particle? Explain your answer. (2 marks)

- 7 An elastic string has natural length 2 metres and modulus of elasticity λ newtons. One end of the string is fixed at the point O , and a particle of mass 20 kg is attached to the other end of the string.

- (a) When in equilibrium the particle is 2.7 metres below O . Show that $\lambda = 560$. (3 marks)
- (b) The particle is now held at O and released from rest. The maximum length of the string in the subsequent motion is L .
- (i) Show that L satisfies the equation

$$5L^2 - 27L + 20 = 0 \quad (5 \text{ marks})$$

- (ii) Find the maximum length of the string. (3 marks)

8 A cyclist moves from rest along a straight horizontal road. At time t seconds, the displacement of the cyclist from his initial position is s metres.

(a) For $0 \leq t \leq 10$,

$$s = \frac{t^4}{400} - \frac{t^3}{10} + \frac{3t^2}{2}$$

(i) Find s when $t = 10$. *(1 mark)*

(ii) Find the velocity of the cyclist when $t = 10$. *(3 marks)*

(iii) Find the acceleration of the cyclist when $t = 10$. *(3 marks)*

(b) For $t \geq 10$ the cyclist moves with a constant velocity, so that

$$s = ht - k$$

where h and k are constants. Find the values of h and k . *(3 marks)*

END OF QUESTIONS