

General Certificate of Education
June 2005
Advanced Level Examination



**MATHEMATICS AND STATISTICS
(SPECIFICATION B)
Unit Mechanics 2**

MBM2

Monday 20 June 2005 Morning Session

In addition to this paper you will require:

- a 12-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 45 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBM2.
- Answer **all** questions.
- Take $g = 9.8 \text{ m s}^{-2}$ unless stated otherwise.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information

- The maximum mark for this paper is 80.
- Mark allocations are shown in brackets.

Advice

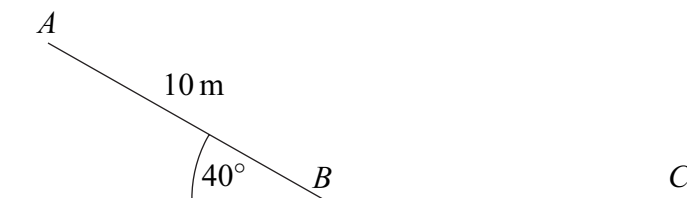
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

- 1 A car accelerates from rest along a straight road for 5 seconds. At time t seconds, its acceleration, $a \text{ m s}^{-2}$, is given by

$$a = t - \frac{t^2}{5}$$

- (a) By integrating, find an expression for the velocity of the car at time t . (3 marks)
- (b) Find the velocity of the car at the end of the 5 second period. (2 marks)
- (c) Find the distance travelled by the car during the 5 second period. (4 marks)
- 2 The diagram shows a simple slide made of two sections AB and BC . Section AB has length 10 metres and is at an angle of 40° to the horizontal. Section BC is horizontal.



A child uses the slide, pushing himself off so that at A his speed is 2 m s^{-1} .

When he reaches B his speed is 6 m s^{-1} .

The mass of the child is 35 kg.

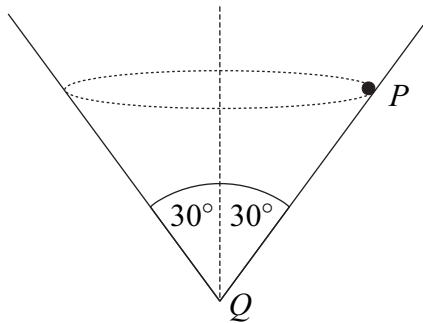
- (a) (i) Calculate the kinetic energy of the child at A . (2 marks)
- (ii) Calculate the gain in kinetic energy as the child moves from A to B . (3 marks)
- (b) Show that the loss in potential energy as the child moves from A to B is 2200 joules, correct to three significant figures. (2 marks)
- (c) As the child moves from A to B , a resistance force acts on him.
- Assume that this force has a constant magnitude F newtons.
- Find F . (3 marks)
- (d) Assume that the same resistance force acts on the child as he moves on BC and that he stops at C . Find the length of BC . (3 marks)

- 3 An aeroplane is circling an airfield while waiting to land. At time t seconds, its position vector, \mathbf{r} metres, with respect to an origin at the airfield is

$$\mathbf{r} = 500 \sin(0.1t)\mathbf{i} + 500 \cos(0.1t)\mathbf{j} + 1000\mathbf{k}$$

The unit vectors \mathbf{i} and \mathbf{j} are directed east and north respectively and the unit vector \mathbf{k} is directed vertically upwards.

- (a) Find the velocity of the aeroplane at time t . (3 marks)
- (b) Hence show that the speed of the aeroplane is 50 m s^{-1} . (4 marks)
- (c) Find the acceleration of the aeroplane. (2 marks)
- (d) The mass of the aeroplane is 8000 kg . Find the magnitude of the resultant force acting on the aeroplane. (3 marks)
- 4 A particle P , of mass m , moves in a horizontal circular path on the inside of a smooth cone with vertex Q . The angle between PQ and the vertical is 30° . The diagram shows the cone and the path of the particle.



- (a) Draw a diagram to show the forces acting on the particle. (1 mark)
- (b) Show that the reaction force on the particle has magnitude $2mg$. (2 marks)
- (c) The speed of the particle is v . Find the radius, r , of the circular path of P in terms of v and g . (4 marks)

5 An elastic string has natural length 2 metres and modulus of elasticity 30 N. One end of the string is fixed at the point O . A particle of mass 0.15 kg is attached to the other end of the string. The particle is pulled down until it is 2.8 metres directly below O and released from rest.

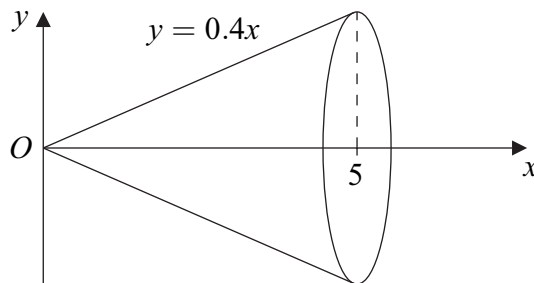
(a) Calculate the elastic potential energy of the string just before the particle is released. (2 marks)

(b) After the particle has been released, it moves upwards.

(i) Show that the speed of the particle as it passes O is 3.02 m s^{-1} , correct to three significant figures. (4 marks)

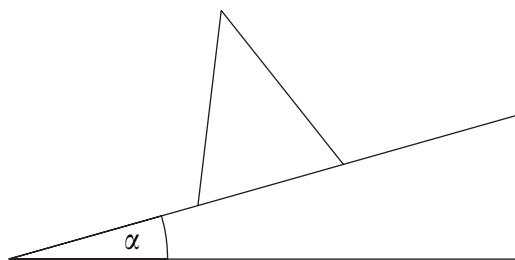
(ii) By considering the maximum height of the particle above O , show that the string does not become taut as the particle moves upwards. (4 marks)

6 A uniform **solid** cone is formed by rotating the finite region bounded by the lines with equations $y = 0.4x$, $y = 0$ and $x = 5$ through 360° about the x -axis. The cone is shown in the diagram below.



(a) Use integration to show that the distance of the centre of mass of the cone from the origin is $\frac{15}{4}$. (5 marks)

(b) The cone is placed on a rough slope, which is at an angle α to the horizontal, as shown in the diagram.



The angle α is gradually increased.

Assuming that the cone does not slide, find the value of α when the cone is on the point of toppling. (5 marks)

7 An elastic string has natural length l and modulus of elasticity λ . One end of the string is fixed and a particle of mass m is attached to the other end of the string.

(a) When the particle hangs vertically in equilibrium, the extension of the string is $\frac{l}{4}$. Show that $\lambda = 4mg$. (2 marks)

(b) The particle is then set into vertical motion and x is the displacement of the particle from the equilibrium position at time t . Assume that the string remains taut during the vertical motion.

(i) Show that $\frac{d^2x}{dt^2} = -\frac{4gx}{l}$. (5 marks)

(ii) Hence find the period of the motion. (2 marks)

8 A stone, of mass m kg, is projected vertically upwards from a catapult. It leaves the catapult travelling at 20 m s^{-1} . As it moves, it is subject to a resistance force of magnitude $0.7mv$ newtons, where $v \text{ m s}^{-1}$ is the speed of the stone at time t seconds.

At time t seconds, the height of the stone is x metres above the point of projection.

(a) Show that, as the stone is moving upwards,

$$v \frac{dv}{dx} = -0.7(v + 14) \quad (2 \text{ marks})$$

(b) Using the identity $\frac{v}{v+14} = 1 - \frac{14}{v+14}$, show that

$$x = \frac{10}{7} \left(20 - v + 14 \ln \left(\frac{v+14}{34} \right) \right) \quad (6 \text{ marks})$$

(c) Hence find the maximum value of x . (2 marks)

END OF QUESTIONS

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