General Certificate of Education
January 2004
Advanced Level Examination

MATHEMATICS AND STATISTICS (SPECIFICATION B)
Unit Mechanics 2

MBM2

ASSESSMENT and QUALIFICATIONS ALLIANCE

Friday 23 January 2004 Morning Session

In addition to this paper you will require:
- a 12-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 45 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBM2.
- Answer **all** questions.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$ unless stated otherwise.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 80.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 A diver has mass 65 kg. She dives from a fixed diving board, which is 6 metres above the level of the water in the pool. When the diver leaves the board, she is travelling vertically upwards and has speed $2\,\mathrm{m\,s^{-1}}$.

Model the diver as a particle. Assume that there are no resistance forces acting on the diver as she moves through the air and that she does not hit the board on the way down.

(a) (i) Calculate the kinetic energy of the diver when she leaves the board. *(2 marks)*

(ii) By using an energy method, calculate the maximum height of the diver above the diving board. *(2 marks)*

(b) (i) Find the kinetic energy of the diver when she hits the water. *(3 marks)*

(ii) Hence calculate the speed of the diver when she hits the water. *(2 marks)*

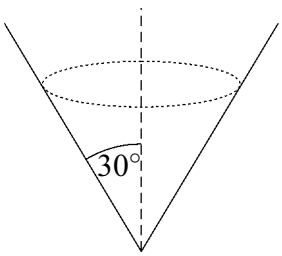
2 A particle moves on a straight line. At time t seconds its acceleration, $a\,\mathrm{m\,s^{-2}}$, is given by

$$a = 20 \sin 4t$$

(a) Initially the particle is at rest. Find an expression for the velocity of the particle at time t. *(4 marks)*

(b) Initially the displacement of the particle from the origin is 0.8 metres. Find an expression for the displacement of the particle at time t. *(4 marks)*

3 A particle, of mass 3 kg, describes a horizontal circular path on the inside surface of a smooth cone, as shown in the diagram.



The radius of the circle is 0.5 metres and the semi-vertical angle of the cone is 30°. The particle moves at a constant speed.

(a) (i) Show that the magnitude of the normal reaction force on the particle is 58.8 N. *(3 marks)*

(ii) Find the speed of the particle. *(4 marks)*

(b) The particle moves on the same cone, but in a horizontal circle of greater radius than before.

(i) What happens to the magnitude of the normal reaction force? *(1 mark)*

(ii) What happens to the speed of the particle? Explain your answer. *(2 marks)*

4 An elastic string has natural length 2 metres and modulus of elasticity λ newtons. One end of the string is fixed at the point O, and a particle of mass 20 kg is attached to the other end of the string.

(a) When in equilibrium the particle is 2.7 metres below O. Show that $\lambda = 560$. *(3 marks)*

(b) The particle is now held at O and released from rest. The maximum length of the string in the subsequent motion is L.

(i) Show that L satisfies the equation

$$5L^2 - 27L + 20 = 0$$ *(5 marks)*

(ii) Find the maximum length of the string. *(3 marks)*

5 A cyclist moves from rest along a straight horizontal road. At time t seconds, the displacement of the cyclist from his initial position is s metres.

(a) For $0 \leqslant t \leqslant 10$,

$$s = \frac{t^4}{400} - \frac{t^3}{10} + \frac{3t^2}{2}$$

(i) Find s when $t = 10$. *(1 mark)*

(ii) Find the velocity of the cyclist when $t = 10$. *(3 marks)*

(iii) Find the acceleration of the cyclist when $t = 10$. *(3 marks)*

(b) For $t \geqslant 10$ the cyclist moves with a constant velocity, so that

$$s = ht - k$$

where h and k are constants. Find the values of h and k. *(3 marks)*

6 A uniform lamina is bounded by the x-axis, and the lines $x = a$ and $y = kx$, where k is a positive constant.

(a) Find the area of the lamina in terms of a and k. *(2 marks)*

(b) Use integration to show that the x-coordinate of the centre of mass of the lamina is $\frac{2a}{3}$. *(4 marks)*

(c) Use integration to find the y-coordinate of the centre of mass of the lamina. *(4 marks)*

7 A particle moves with simple harmonic motion on a straight line between two points A and B, which are 0.4 metres apart. The maximum speed of the particle is $10\,\mathrm{m\,s^{-1}}$.

(a) Show that the period of the motion is $\frac{\pi}{25}$ seconds. *(4 marks)*

(b) Find the speed of the particle when it is 0.04 metres from A. *(3 marks)*

(c) The distance, s, of the particle from A at time t is given by

$$s = p - q\cos(\omega t)$$

where ω, p and q are constants.

(i) State the values of ω and q. *(2 marks)*

(ii) When $t = 0$ the particle is at A. Find the value of p. *(2 marks)*

8 A stone, of mass 0.1 kg, is projected vertically upwards from a catapult at a speed of $12\,\mathrm{m\,s^{-1}}$. As the stone rises it is acted on by gravity and air resistance. When the stone is moving at $v\,\mathrm{m\,s^{-1}}$, the air resistance has magnitude $\frac{v^2}{200}$ newtons.

(a) The height of the stone above the level of projection at time t seconds is x metres. Show that while the stone is rising

(i) $v\frac{\mathrm{d}v}{\mathrm{d}x} = -\left(9.8 + \frac{v^2}{20}\right)$ *(2 marks)*

(ii) $10\ln\left(9.8 + \frac{v^2}{20}\right) + x = 10\ln 17$ *(6 marks)*

(b) Find the maximum height of the stone. *(3 marks)*

END OF QUESTIONS