GCE 2005
January Series

ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark Scheme

Mathematics and Statistics B
(MBM2)

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Key to Mark Scheme

M ..... mark is for ..... method
m ..... mark is dependent on one or more M marks and is for ..... method
A ..... mark is dependent on M or m marks and is for ..... accuracy
B ..... mark is independent of M or m marks and is for ..... method and accuracy
E ..... mark is for ..... explanation
√ **or ft or F** ..... follow through from previous incorrect result
CAO ..... correct answer only
AWFW ..... anything which falls within
AWRT ..... anything which rounds to
AG ..... answer given
SC ..... special case
OE ..... or equivalent
A2,1 ..... 2 or 1 (or 0) accuracy marks
-*x* EE ..... deduct *x* marks for each error
NMS ..... no method shown
PI ..... possibly implied
SCA ..... substantially correct approach
c ..... candidate
SF ..... significant figure(s)
DP ..... decimal place(s)

Abbreviations used in Marking

MC – *x* ..... deducted *x* marks for mis-copy
MR – *x* ..... deducted *x* marks for mis-read
ISW ..... ignored subsequent working
BOD ..... given benefit of doubt
WR ..... work replaced by candidate
FB ..... formulae booklet

Application of Mark Scheme

No method shown:
Correct answer without working ..... mark as in scheme
Incorrect answer without working ..... zero marks unless specified otherwise

More than one method/choice of solution:

2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only

Crossed out work — do not mark unless it has not been replaced

Alternative solution using a correct or partially correct method — award method and accuracy marks as appropriate

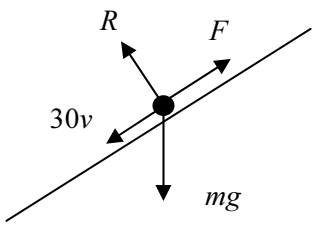
Mathematics and Statistics B Mechanics 2 MBM2 January 2005

Question Number and Part	Solution	Marks	Total	Comments
1(a)	$\mathbf{v} = -4e^{-t}\mathbf{i} + (6 - 3e^{-t})\mathbf{j}$	M1		Differentiating position vector
		A1		Correct velocity
	$t = 0$			
	$\mathbf{v} = -4\mathbf{i} + 3\mathbf{j}$	A1	3	**ag** Substituting $t = 0$ to obtain initial velocity
(b)	$\mathbf{a} = 4e^{-t}\mathbf{i} + 3e^{-t}\mathbf{j}$	M1		Differentiating velocity
		A1	2	Correct acceleration
(c)	$\mathbf{a} = 4\mathbf{i} + 3\mathbf{j}$	M1		Finding acceleration when $t = 0$
	$a = \sqrt{4^2 + 3^2} = 5$	A1	2	Correct magnitude
(d)	$\mathbf{v} \to 0\mathbf{i} + 6\mathbf{j}$	B1		For **i** component
		B1	2	For **j** component
	Total		**9**	
2(a)	$EPE = \frac{1}{2} \times \frac{40}{2} \times 3^2 = 90$ J	M1		Finding EPE
		A1	2	**ag** Correct EPE from correct working
(b)	$90 = \frac{1}{2} \times 5v^2$	M1		Use of EPE = KE
		A1		Correct equation
	$v^2 = 36$			
	$v = 6$	A1	3	**ag** Correct speed from correct working
(c)	$EPE = \frac{1}{2} \times \frac{40}{2} \times 1^2 = 10$ J	M1		Finding EPE 3 metres from O
		A1		Correct EPE
	$90 - 10 = \frac{1}{2} \times 5v^2$	M1		Using EPE lost = KE
		A1		Correct equation
	$v^2 = 32$			
	$v = 5.66$ ms^{-1} (to 3 sf)	A1	5	Correct speed
	Total		**10**	

MBM2(cont)

Question Number and Part	Solution	Marks	Total	Comments
3(a)	$P = 2000$	B1		Correct value for P
	$Q = 100$	B1	2	Correct value for Q
(b)	$a = -\frac{F}{1000} = \frac{t}{10} - 2$	M1		Use of $F = ma$
		A1	2	**ag** Correct expression from correct working
(c)	$v = \frac{t^2}{20} - 2t + c$	M1		Integrating acceleration to give velocity
		A1		Correct velocity with or without c
	$0 = \frac{20^2}{20} - 2\times 20 + c$	M1		Finding c
	$c = 20$			
	$v = \frac{t^2}{20} - 2t + 20$	A1√	4	Correct expression for the velocity ft incorrect constants from (b)
(d)	$s = \int_0^{20} \frac{t^2}{20} - 2t + 20 \, \mathrm{d}t$	M1		Integrating velocity
	$= \left[\frac{t^3}{60} - t^2 + 20t\right]_0^{20}$	A1		Correct integral
		A1		Correct limits/value of c
		M1		Finding distance by substituting limits
	$= 133$ m	A1	5	Correct distance
	Total		**13**	

MBM2(cont)

Question Number and Part	Solution	Marks	Total	Comments
4(a)		B1	1	Correct force diagram
(b)	$F = 1500g\cos 85° + 300$	M1		Finding F
		A1		Correct F
	$P = (1500g\cos 85° + 300)\times 10$	M1		Use of $P = Fv$
	$= 15800$ W (to 3 sf)	A1	4	**ag** Correct answer from correct working
(c)	$F = 1500g\cos 85° + 30v$	M1		F in terms of v
		A1		Correct expression for F
	$35000 = v(1500g\cos 85° + 30v)$	m1		Using $P = Fv$ to obtain a quadratic
	$0 = 30v^2 + 1281v - 35000$	A1		Correct quadratic
	$v = \dfrac{-1281 \pm \sqrt{1281^2 + 4\times 30\times 35000}}{2\times 30}$	m1		Solving quadratic equation
	$= 18.9$ or -61.6			
	Max Speed $= 18.9$ ms^{-1}	A1	6	Correct speed
	Total		**11**	
5(a)	$a = 0.6\times 10^2 = 60$ ms^{-2}	M1		Use of $a = r\omega^2$
		A1	2	Correct acceleration Allow ± 60
(b)	$R = 0.05\times 60 = 3$ N	M1		Finding product of mass and acceleration
		A1√	2	Correct R Follow through incorrect a
(c)	$R - 0.05\times 9.8 = 0.05\times 60$	M1		Equation of motion at lowest point
		A1		Correct equation
	$R = 3.49$ N	A1√	3	Correct R Follow through incorrect a
	Total		**7**	

MBM2(cont)

Question Number and Part	Solution	Marks	Total	Comments
6(a)	$2\times 9.8 = \frac{\lambda}{0.5}\times 0.2$	M1		Equilibrium considered to form equation in λ
	$\lambda = \frac{9.8}{0.2} = 49$ N	A1 A1	3	Correct equation Correct λ
(b)	$T = \frac{49x}{0.5} + 2g$	M1 A1		Equation for tension with two terms. Correct equation.
	$2\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 2g - (98x + 2g)$	M1		Use of $F = m\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\frac{98}{2}x = -49x$	A1 A1	5	Correct equation **ag** Correct result from correct working
(c)	Period $= \frac{2\pi}{\sqrt{49}}$	M1 A1		Finding period Correct period
	$t = \frac{1}{4}\times\frac{2\pi}{7} = \frac{\pi}{14} = 0.224$ seconds	M1 A1	4	Dividing period by 4 Correct time
	Total		**12**	
7(a)	$V = \pi\int_0^2 2 - x\ \mathrm{d}x$	M1		Use of $\int y^2\mathrm{d}x$
	$= \pi\left[2x - \frac{x^2}{2}\right]$	A1		Correct expression for the volume
	$= \pi(4-2) = 2\pi$	A1	3	**ag** Correct volume from correct working
(b)	$2\pi\bar{x} = \pi\int_0^2 2x - x^2\mathrm{d}x$	M1		Use of $\int xy^2\mathrm{d}x$
	$= \pi\left[x^2 - \frac{x^3}{3}\right]_0^2$	A1		Correct expression containing $\bar{x}$
	$= \pi\left(4 - \frac{8}{3}\right)$	M1		Evaluating integral
	$\bar{x} = \frac{2}{3}$	A1	4	Correct final answer
(c)	$\tan\alpha = \frac{\frac{2}{3}}{\sqrt{2}}$	B1 M1 A1		Use of $\sqrt{2}$ Use of tan to find angle Correct expression for tan
	$\alpha = 25.2°$	A1√	4	Correct angle Follow through from part (b)
	Total		**11**	

MBM2(cont)

Question Number and Part	Solution	Marks	Total	Comments
8	$mv\dfrac{\mathrm{d}v}{\mathrm{d}x}=-kv^2$	M1		Forming a differential equation using $v\dfrac{\mathrm{d}v}{\mathrm{d}x}$
	$\int\dfrac{1}{v}\mathrm{d}v=\int-\dfrac{k}{m}\mathrm{d}x$	M1		Use of integration to obtain a $\ln v$ term
	$\ln v=-\dfrac{k}{m}x+c$	A1		Correct integral with or without c
	$x=0, v=U\Rightarrow c=\ln U$	M1		Finding value of c
	$\ln v=-\dfrac{k}{m}x+\ln U$	A1		Correct value of c
	$\ln\left(\dfrac{v}{U}\right)=-\dfrac{k}{m}x$	M1		Making v the subject
	$\dfrac{v}{U}=\mathrm{e}^{-\frac{k}{m}x}$			
	$v=U\mathrm{e}^{-\frac{k}{m}x}$	A1	7	Correct expression for v
	Total		**7**	
	TOTAL		**80**	