General Certificate of Education
June 2005
Advanced Level Examination

**MATHEMATICS AND STATISTICS** **MBD2**
**(SPECIFICATION B)**
**Unit Discrete 2**

Monday 20 June 2005 Morning Session

**In addition to this paper you will require:**
- a 12-page answer book;
- the AQA booklet of formulae and statistical tables;
- an insert for use in Question 5 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 45 minutes

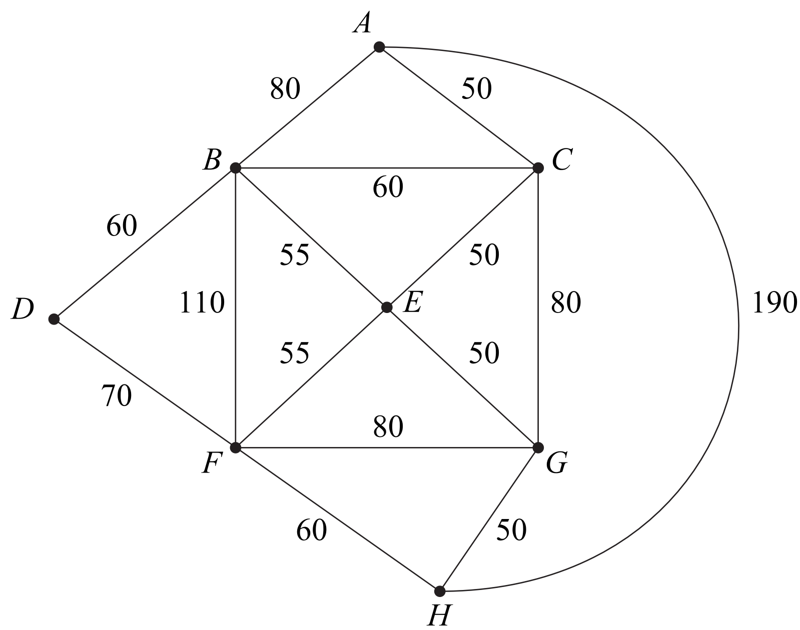
**Instructions**
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBD2.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Fill in the boxes at the top of the insert. Make sure that you attach this insert to your answer book.

**Information**
- The maximum mark for this paper is 80.
- Mark allocations are shown in brackets.

**Advice**
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

**1** Eight alpine huts $A - H$ are connected by footpaths, as shown in the following network. The numbers denote the lengths of the paths, in metres.




The total length of all the footpaths is 1100 metres.

(a) A snowplough, starting at $A$, needs to travel along each of the footpaths at least once, and finish back at $A$.

(i) Explain how you know that it will be necessary for the snowplough to go along some footpaths more than once. *(1 mark)*

(ii) Apply the Chinese postperson algorithm to find the shortest distance which the snowplough will have to travel. (You may find the shortest distances between pairs of vertices by inspection.) *(4 marks)*

(iii) Give an example of a suitable shortest route. *(3 marks)*

(b) The caretaker of the huts wants to start at $A$, pass each hut exactly once, and finish back at $A$.

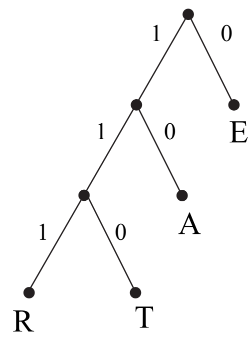
(i) Use a nearest neighbour approach to find a suitable route of length 470 metres. *(3 marks)*

(ii) Explain why, if the caretaker uses footpath $AH$, the total length of his route will be more than $(190 + (7 \times 50))$ metres. *(1 mark)*

(iii) Show that, if the caretaker does not use footpath $AH$, his route must include footpaths $AB$, $AC$, $DB$, $DF$, $HF$ and $HG$. Hence show that the caretaker's route which you found in part (b)(i), or its reverse, is the caretaker's only route which does not use footpath $AH$. *(2 marks)*

(iv) Deduce that the caretaker's route which you found in part (b)(i) is the shortest possible. *(2 marks)*

**2** The tree for a Huffman code of four letters is shown below:

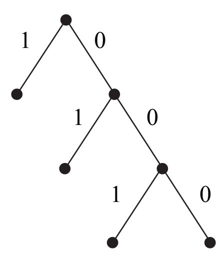


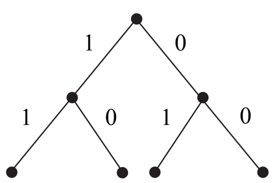

So, for example, T encodes as 110.

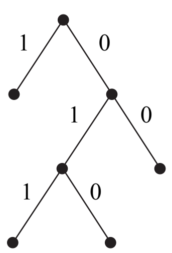
(a) Encode RATE in the above code. *(1 mark)*

(b) Decode 111010111 in the above code. *(1 mark)*

(c) Five trees are suitable for Huffman codes of four letters. One is shown above and three more are shown below:




Draw the fifth tree. *(2 marks)*

(d) The letters A, E, R, T are allocated to a tree for a Huffman code of four letters so that TREAT encodes as 00011010100.

(i) What does T encode as? Give a reason. *(2 marks)*

(ii) Draw the tree and the allocation of letters to it. *(2 marks)*

**3** (a) Show that the recurrence relation

$$u_n - u_{n-1} - 2u_{n-2} = 1$$

has general solution

$$u_n = A.2^n + B.(-1)^n - \tfrac{1}{2}$$ *(5 marks)*

(b) The sequence of integers $u_0, u_1, u_2, u_3, \ldots$ is given by

$$u_0 = 0, \; u_1 = 1 \quad \text{and} \quad u_n - u_{n-1} - 2u_{n-2} = 1, \; n > 1$$

(i) Use your answer from part (a) to find a formula for the integer $u_n$. *(3 marks)*

(ii) Deduce that $u_n$ is given by

$$u_n = \begin{cases} \dfrac{2^{n+1}}{3} - \dfrac{2}{3} & \text{if } n \text{ is even} \\[2ex] \dfrac{2^{n+1}}{3} - \dfrac{1}{3} & \text{if } n \text{ is odd} \end{cases}$$

and hence that $u_n$ is the largest integer less than $\dfrac{2^{n+1}}{3}$. *(3 marks)*

**4** A linear binary code has eight codewords, five of which are

001110
011111
110011
100010
101100

(a) Find the other three codewords. *(3 marks)*

(b) In each codeword:

the sum of the entries in the first, third and fifth positions is 0;
the second and sixth entries are the same.

Find a third linear property involving the fourth entry and hence write down a parity check matrix for the code. *(3 marks)*

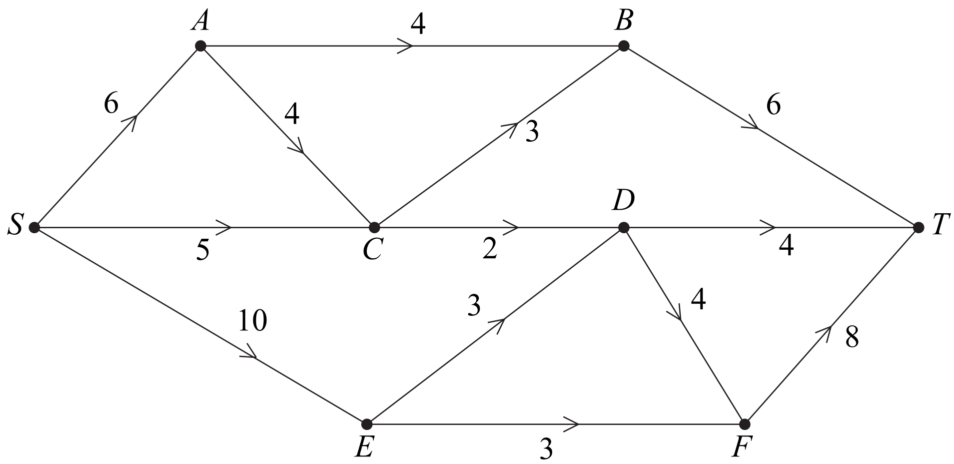
(c) A message is received as

010001101010

and it contains at most one error. Use the parity check matrix to correct the message. *(3 marks)*

**5** [**Figure 1**, printed on the insert, is provided for use in answering part (a) of this question.]

The network shows a system of pipes and their capacities:




(a) Starting from a zero flow, use flow-augmenting paths on **Figure 1** to find a maximum flow from $S$ to $T$. *(5 marks)*

(b) Find a cut which confirms that your flow in (a) is a maximum. *(2 marks)*

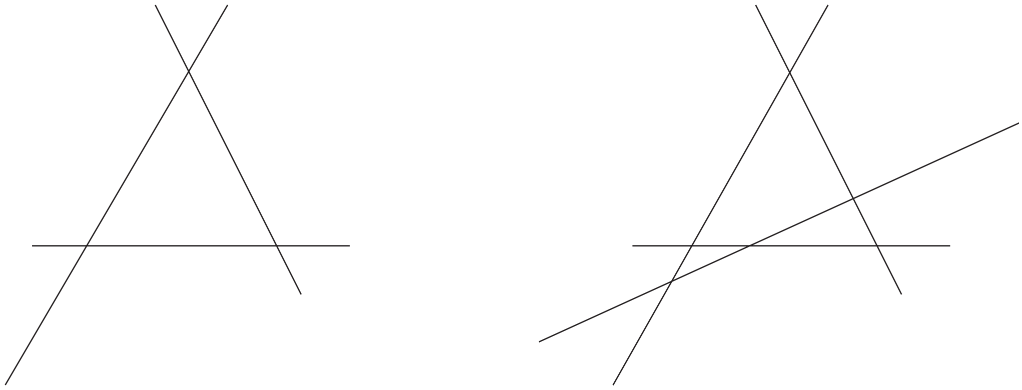
(c) (i) Show that in any flow from $S$ to $T$ there can be at most a flow of 6 in the pipe $SE$. *(1 mark)*

(ii) Show that, if the pipe $AC$ is saturated during a flow from $S$ to $T$, there can be at most a flow of 1 in the pipe $SC$. *(2 marks)*

(iii) Deduce that in any maximum flow from $S$ to $T$ the pipe $AC$ will not be saturated. *(2 marks)*

**TURN OVER FOR THE NEXT QUESTION**

**6** Three non-parallel lines make three intersection points, as shown on the left below. When a fourth non-parallel line is added, passing through none of the existing intersection points, it creates three new intersection points, as shown on the right below.



Let $p_n$ denote the total number of intersection points made by $n$ non-parallel lines, with no line passing through an existing intersection point.

(a) State the values of $p_2$, $p_3$ and $p_4$. *(2 marks)*

(b) If a fifth line is added to the right-hand figure above, meeting each of the existing lines but passing through none of the existing intersection points, how many new intersection points will be created? Deduce that

$$p_5 = p_4 + 4$$

and explain briefly why

$$p_n = p_{n-1} + (n - 1)$$ *(2 marks)*

(c) Verify that $p_n = \frac{1}{2}n(n-1)$ is the solution of the recurrence relation

$$p_1 = 0, \quad p_n = p_{n-1} + (n-1), \quad n > 0$$ *(3 marks)*

**7** A drug company produces three types of popular painkiller: Xtrafast, Yourelax and Zizz. Each of these is a combination of three key ingredients: aspirin, bupro and codeine. The table below shows the numbers of units of each of the three ingredients required in making batches of the three painkillers. It also shows the amounts of the ingredients available and the profit on each batch of painkillers.

| | **aspirin** | **bupro** | **codeine** | **profit** |
|---|---|---|---|---|
| **Xtrafast** | 1 | 1 | 3 | £15 |
| **Yourelax** | 2 | 1 | 6 | £10 |
| **Zizz** | 1 | 1 | 2 | £10 |
| **Amount available** | 60 | 55 | 140 | |

The company wishes to maximise its profits.

Let $x$ be the number of batches of Xtrafast made, $y$ the number of Yourelax and $z$ the number of Zizz.

(a) State the problem as a linear programming problem, writing down the objective function and the full set of inequalities. *(3 marks)*

(b) Copy and complete the following initial tableau for the simplex method when applied to this problem:

| $P$ | $x$ | $y$ | $z$ | $s$ | $t$ | $u$ | |
|---|---|---|---|---|---|---|---|
| 1 | −15 | −10 | −10 | 0 | 0 | 0 | 0 |
| 0 | 1 | 2 | 1 | 1 | 0 | 0 | 60 |
| . | . | . | . | . | . | . | . |
| . | . | . | . | . | . | . | . |

*(2 marks)*

(c) Perform one iteration of the simplex method by increasing $z$. *(5 marks)*

(d) Perform a second iteration of the simplex method. *(4 marks)*

(e) State the maximum profit possible and how many batches of Xtrafast, Yourelax and Zizz should be made in order to obtain that profit.

Explain why some people might not be happy with this solution. *(3 marks)*

**END OF QUESTIONS**

**THERE ARE NO QUESTIONS PRINTED ON THIS PAGE**

| Surname | | Other Names | |
|---|---|---|---|
| Centre Number | | Candidate Number | |
| Candidate Signature | | | |

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Insert for use in Question 5.

Fill in the boxes at the top of this page.

Attach this insert securely to your answer book.

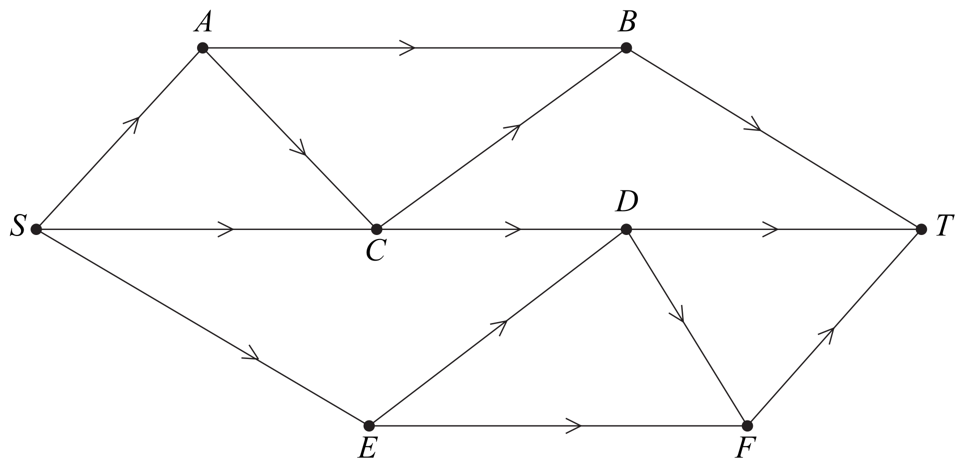


**Figure 1**

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