

General Certificate of Education
January 2005
Advanced Level Examination



**MATHEMATICS AND STATISTICS
(SPECIFICATION B)
Unit Discrete 2**

MBD2

Tuesday 25 January 2005 Morning Session

In addition to this paper you will require:

- a 12-page answer book;
- the AQA booklet of formulae and statistical tables;
- an insert for use in Question 2 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 45 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBD2.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Fill in the boxes at the top of the insert. Make sure that you attach this insert to your answer book.

Information

- The maximum mark for this paper is 80.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 The sequence p_1, p_2, p_3, \dots satisfies the recurrence relation

$$p_1 = 0, \quad p_n = 2p_{n-1} + 1, \quad n > 1$$

- (a) Solve the recurrence relation to derive an expression for p_n in terms of n . Simplify your answer as far as possible. *(4 marks)*
- (b) Calculate the values of p_4 and p_5 . *(1 mark)*
- (c) In fact p_n equals the number of ways of splitting a set of n items into two separate parts. So, for example, $p_3 = 3$ since the set $\{a,b,c\}$ can be split into two parts in three ways, namely

$$\{a\} \text{ and } \{b,c\} \quad \{a,b\} \text{ and } \{c\} \quad \{a,c\} \text{ and } \{b\}$$

List the p_4 different ways of splitting the set $\{a,b,c,d\}$ into two parts. *(3 marks)*

2 [Figure 1, printed on the insert, is provided for use in answering part (a)(ii) of this question.]

The towns P, Q, R, S, T, U, V and W are all linked to each other by direct roads whose lengths, in miles, are shown in the following table:

	P	Q	R	S	T	U	V	W
P	—	5	4	3	4	5	4	6
Q	5	—	7	5	3	5	8	8
R	4	7	—	6	8	6	3	5
S	3	5	6	—	4	6	9	6
T	4	3	8	4	—	7	8	6
U	5	5	6	6	7	—	8	3
V	4	8	3	9	8	8	—	8
W	6	8	5	6	6	3	8	—

- (a) An estate agent wants to start at the town P and visit each of the other towns once before returning to P .
- Use the nearest neighbour algorithm to find one possible route for the estate agent and state its total length. (4 marks)
 - On **Figure 1**, ignore the town P and find the length of a minimum connector of the towns Q, R, S, T, U, V and W . (4 marks)
 - Deduce a lower bound for the estate agent's route and comment on its significance to your answer to part (a)(i). (3 marks)
- (b) The estate agent would then like to look at the houses on each of the roads joining the towns. The estate agent starts at P , covers each of the roads at least once, and finishes at P .
- Explain why it will be necessary to cover at least four of the roads more than once. (2 marks)
 - Given that the total length of all the roads is 160 miles, explain how you know that all the roads can be covered with a round trip of 172 miles. (3 marks)

3 Oxfield College gives each of its students a 9-digit registration number

$$x_1x_2x_3x_4x_5x_6x_7x_8x_9$$

where the four digits $x_1x_2x_3x_4$ show the year in which the student started and the final digit x_9 is chosen so that

$$x_1 + 3x_2 + x_3 + 3x_4 + x_5 + 3x_6 + x_7 + 3x_8 + x_9$$

is divisible by 10.

- (a) Explain the purpose of the last digit. *(1 mark)*
- (b) Verify that 200432298 is a correct registration number. *(1 mark)*
- (c) A student who started in 2004 writes his registration number as 200432299 but he has written one of the digits incorrectly. Find the five possible correct registration numbers. *(4 marks)*
- (d) Another student incorrectly writes her registration number as 200432296 by writing two adjacent digits in the wrong order. What is her correct number? *(2 marks)*
- (e) If each student is to have a different registration number, what is the maximum number of students that the college can accept in any one year? *(2 marks)*
- (f) Show that in any correct registration number the sum of the nine digits is even. *(2 marks)*

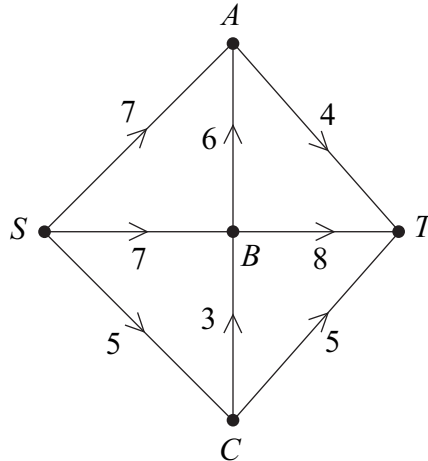
- 4 A linear programming problem consists of maximising an objective function P in three variables x, y and z . The simplex method is used to solve the problem and several iterations of the method lead to the following tableau:

P	x	y	z	s	t	u	v	
1	-2	0	0	2	0	0	1	60
0	-2	0	1	1	0	0	-1	15
0	1	0	0	2	1	0	2	25
0	2	0	0	-1	0	1	1	40
0	1	1	0	0	0	0	4	30

- (a) What is the name given to the variables s, t, u and v ? *(1 mark)*
- (b) Other than $x \geq 0, y \geq 0$ and $z \geq 0$, how many inequalities are involved in the problem? *(1 mark)*
- (c) Apply one further iteration of the simplex method to the above tableau. *(5 marks)*
- (d) Explain why your new tableau solves the original problem. State the maximum value of P and the values of x, y and z for which that maximum is reached. *(3 marks)*
- (e) State the values of s, t, u and v at the optimal point. How many of the inequalities from the original problem still have some slack? *(2 marks)*

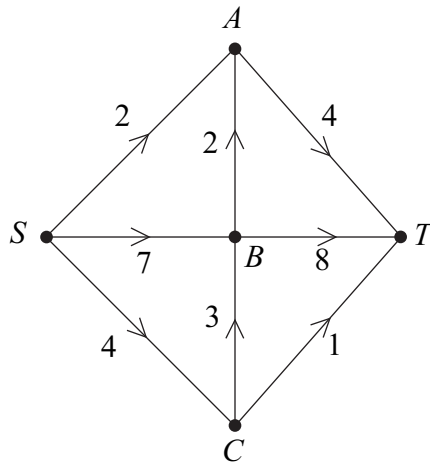
TURN OVER FOR THE NEXT QUESTION

5 The diagram shows the capacities of the arcs of a network.



(a) Find a cut of 16 in the network above. (2 marks)

(b) The next diagram shows a flow of 13 from S to T in the same network.



(i) Find a flow-augmenting path from S to T which uses just one other vertex and which raises the given flow of 13 to a flow of 14. (2 marks)

(ii) Find a further flow-augmenting path from S to T which raises the flow from 14 to 16. (2 marks)

(c) How do you know that the flow of 16 is the maximum possible in this network? (1 mark)

(d) The capacity of one arc of the network is to be increased in order to enable a flow of 19 from S to T . State which arc should be chosen and give a reason. (3 marks)

- 6 (a) Find the general solution of the recurrence relation

$$u_n + 2u_{n-1} - 3u_{n-2} = 0 \quad (4 \text{ marks})$$

- (b) Given that k is a constant and $u_n = kn$ satisfies the recurrence relation

$$u_n + 2u_{n-1} - 3u_{n-2} = 16$$

find the value of k . (3 marks)

- (c) Write down the general solution of the recurrence relation

$$u_n + 2u_{n-1} - 3u_{n-2} = 16 \quad (1 \text{ mark})$$

- (d) Given that $u_0 = u_1 = 1$, find the solution of the recurrence relation

$$u_n + 2u_{n-1} - 3u_{n-2} = 16 \quad (4 \text{ marks})$$

- 7 A linear binary code has parity check matrix

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

So, for example, 0001111 and 0011000 are two of the codewords.

- (a) Calculate the other six codewords. (4 marks)
- (b) Find the Hamming distance of the code and state the number of errors in a word which the code can detect. (2 marks)
- (c) Given that the message

11001000110111

contains at most one error in each codeword, use the parity check matrix to find the two possible correct messages. (4 marks)

END OF QUESTIONS

Surname						Other Names				
Centre Number						Candidate Number				
Candidate Signature										

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Insert for use in Question 2.

Fill in the boxes at the top of this page.

Attach this insert securely to your answer book.

	<i>P</i>	<i>Q</i>	<i>R</i>	<i>S</i>	<i>T</i>	<i>U</i>	<i>V</i>	<i>W</i>
<i>P</i>	—	5	4	3	4	5	4	6
<i>Q</i>	5	—	7	5	3	5	8	8
<i>R</i>	4	7	—	6	8	6	3	5
<i>S</i>	3	5	6	—	4	6	9	6
<i>T</i>	4	3	8	4	—	7	8	6
<i>U</i>	5	5	6	6	7	—	8	3
<i>V</i>	4	8	3	9	8	8	—	8
<i>W</i>	6	8	5	6	6	3	8	—

Figure 1

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