General Certificate of Education
January 2004
Advanced Level Examination

MATHEMATICS AND STATISTICS (SPECIFICATION B) Unit Discrete 2

MBD2

ASSESSMENT and QUALIFICATIONS ALLIANCE

Friday 23 January 2004 Morning Session

In addition to this paper you will require:
- a 12-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 45 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBD2.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 80.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 A sequence $u_1, u_2, u_3, \ldots$ is given by the recurrence relation

$$u_1 = 3 \quad \text{and} \quad u_n = \tfrac{1}{2}u_{n-1} + 1, \quad n > 1$$

(a) Solve the recurrence relation to find an explicit formula for u_n in terms of n. *(4 marks)*

(b) State the value to which u_n tends in the long run. *(1 mark)*

2 In the Morse code each letter is represented by a sequence of dots and dashes. For example, the letters A–E are coded as follows:

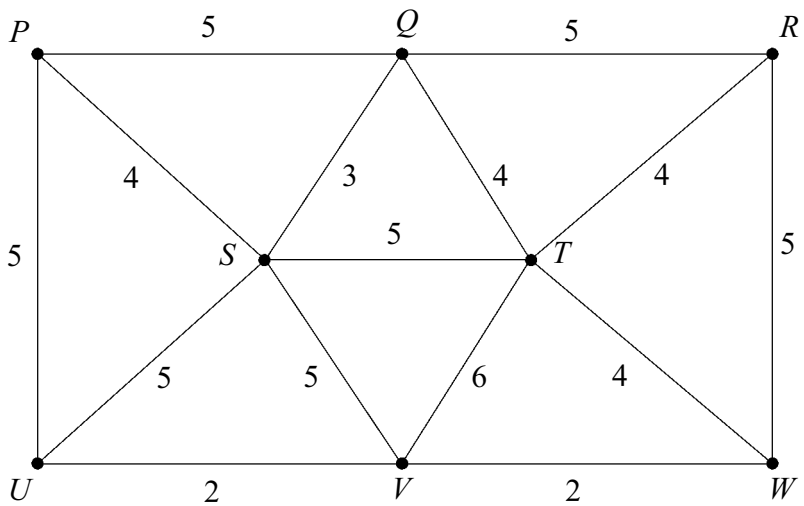
A	B	C	D	E
• —	— • • •	— • — •	— • •	•

(a) Write out the Morse code for the word DEED. *(2 marks)*

(b) Show that your answer to part (a), when read without pauses between letters, can be decoded as another word. *(2 marks)*

(c) It is proposed to use Morse code as the basis of an electronic binary code with 0 replacing each dot and 1 replacing each dash. Give a reason why this is impractical. *(1 mark)*

3 The network shows eight Chinese railway stations *P*–*W* and the lengths, in miles, of the tracks linking them:



(a) The Chinese Trackrail inspector wishes to travel by rail and check the whole network by starting at *P*, travelling along each track at least once, and ending back at *P*.

(i) Explain why it will be necessary to repeat at least three tracks. *(2 marks)*

(ii) The inspector wishes to minimise the distance he has to travel. Determine which tracks he must repeat, given that they will include *PS* or *PU*. *(5 marks)*

(b) Find the length of the minimum connector of the network and illustrate this minimum connector as a tree. *(4 marks)*

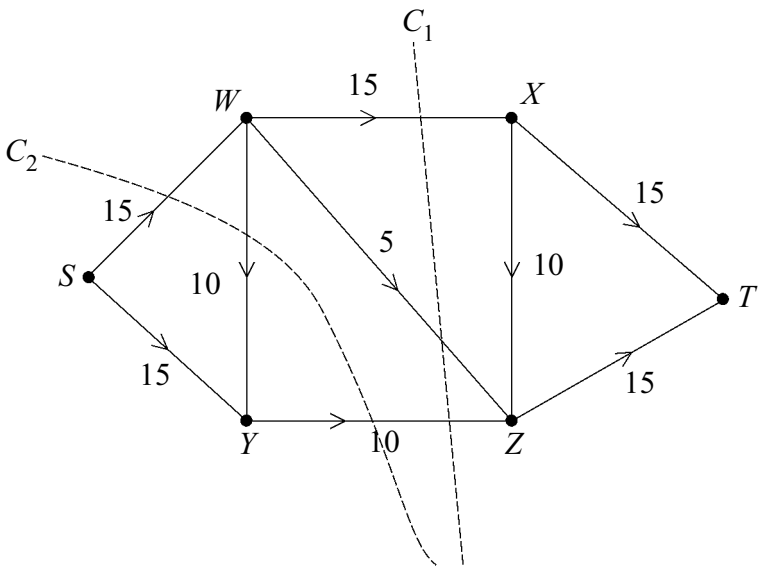
(c) A train-spotter has to travel exactly 25 miles to get from his home to each of the stations *P*–*W*. He wishes to travel from home to a station, then by train to another station and another and so on until he has visited all the stations once. Then he will return home.

(i) Use your answer to part (b) to show that his round trip must be at least 73 miles in length. *(2 marks)*

(ii) Explain why a route of 73 miles is not possible and, by adapting your minimum connector, find a round trip of length 74 miles for the train-spotter. *(3 marks)*

TURN OVER FOR THE NEXT QUESTION

4 In the network shown below, S is the source, T is the sink, and the number on each arc indicates its capacity:

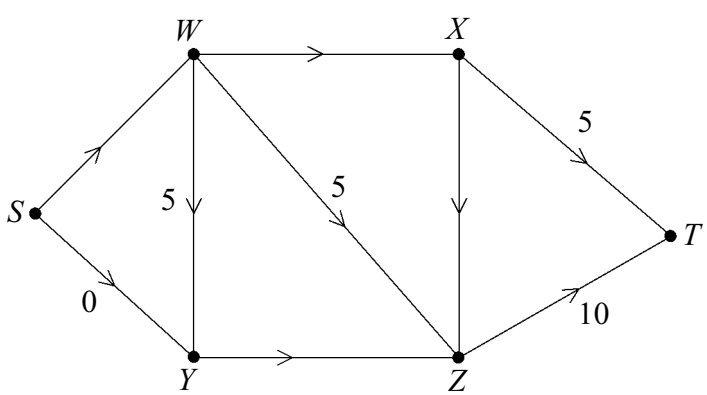


(a) Find the value of:

(i) the cut C_1; *(1 mark)*

(ii) the cut C_2. *(1 mark)*

(b) The diagram below shows a flow from S to T in which the individual flows in four of the arcs have been omitted.



Calculate the flows in YZ, XZ, WX and SW. *(3 marks)*

(c) Find two flow-augmenting paths, each of value 5, which together can be added to the flow illustrated in part (b). *(4 marks)*

(d) Explain how you know that no additional flow-augmenting paths exist. *(2 marks)*

5 A linear binary code has 8 codewords which include:

11000
11101
00110

(a) List the other 5 codewords. *(3 marks)*

(b) (i) Calculate the Hamming distance of the code. *(2 marks)*

(ii) Show that the code can detect single errors. *(1 mark)*

(c) By considering the received message 01011, show that the code has no error-correcting capability. *(2 marks)*

(d) By considering the first two entries in each codeword and the sum of the entries, or otherwise, write down a parity-check matrix for this code. *(2 marks)*

(e) To improve the error-correcting capabilities of the code, each of its codewords $x_1x_2x_3x_4x_5$ is replaced by a new codeword $x_1x_2x_3x_4x_5x_1x_2x_3x_4x_5$.

(i) State the Hamming distance of this new code. *(2 marks)*

(ii) How many errors in a codeword can now be corrected? *(1 mark)*

TURN OVER FOR THE NEXT QUESTION

6 Prudence put £460 into a new Building Society savings account on 1st January 2000 and has left it there untouched. On every subsequent 1st January the Society adds interest to the account, calculated as 5% of the current balance. In addition, after an account has been open at least two years, the Society pays a loyalty bonus on each 1st January. This is calculated as $5\frac{1}{2}\%$ of the balance that was held on the previous 1st January provided that no withdrawals have been made in the meantime.

Let P_n be the amount, in pounds, in the account after any interest payments are added on 1st January n years after the year 2000 (and assuming that no withdrawals are made).

(a) State the values of P_0 and P_1. *(2 marks)*

(b) Show that P_n satisfies the recurrence relation

$$200P_n - 210P_{n-1} - 11P_{n-2} = 0, \qquad n \geqslant 2$$ *(3 marks)*

(c) Find the general solution of the recurrence relation given in part (b). *(5 marks)*

(d) Use the initial conditions from part (a) to find a formula for P_n. *(4 marks)*

7 A furniture company can produce stools, armchairs and settees, each of which requires components A, B and C. The table below shows the numbers of components needed, the numbers of components available, and the profit on each item of furniture.

	Component A	**Component B**	**Component C**	**Profit per item**
Stool	2	1	2	£20
Armchair	1	1	3	£10
Settee	2	1	3	£30
Number available	110	60	140	

The company wishes to maximise its profits. Let x be the number of stools made, y the number of armchairs made, and z the number of settees made.

(a) State the problem as a linear programming problem, writing down the objective function and the full set of inequalities. *(3 marks)*

(b) Copy and complete the following initial tableau for the simplex method when applied to this problem.

P	***x***	***y***	***z***	***s***	***t***	***u***	
1	−20	−10	−30	0	0	0	0
0	2	1	2	1	0	0	110
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.

(2 marks)

(c) Perform one iteration of the simplex method by increasing x. *(5 marks)*

(d) Perform a second iteration of the simplex method. *(3 marks)*

(e) State how many stools, armchairs and settees should be made in order to maximise the company's profits. Comment on the practicality of this solution. *(3 marks)*

END OF QUESTIONS