General Certificate of Education
June 2005
Advanced Subsidiary Examination

MATHEMATICS AND STATISTICS (SPECIFICATION B) Unit Discrete 1

MBD1

Wednesday 25 May 2005 Afternoon Session

In addition to this paper you will require:
- a 12-page answer book;
- the AQA booklet of formulae and statistical tables;
- an insert for use in Questions 1 and 6 (enclosed);
- a ruler.

You may use a graphics calculator.

Time allowed: 1 hour 45 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBD1.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Fill in the boxes at the top of the insert. Make sure that you attach the insert to your answer book.

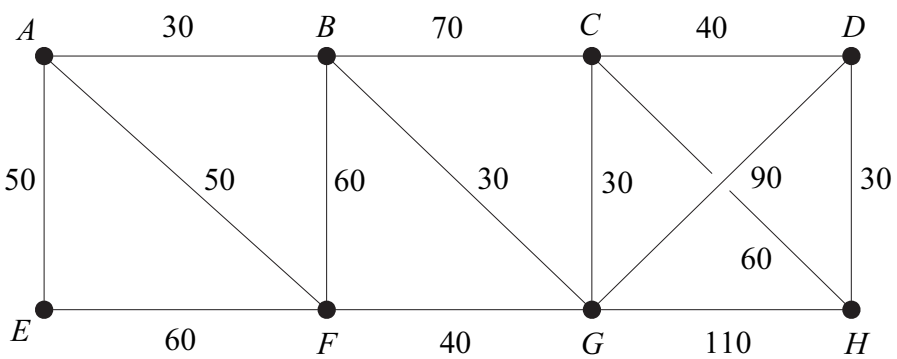
Information
- The maximum mark for this paper is 80.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 [**Figure 1**, printed on the insert, is provided for use in answering part (a) of this question.]

Bryanair operates flights between eight cities $A-H$. The routes and the fares, in pounds, are shown in the following network:



(a) (i) Use Dijkstra's algorithm on **Figure 1** to find the minimum cost of getting from A to H using these routes. Show all your working at each vertex. *(5 marks)*

(ii) State which routes should be used in order to minimise the cost of getting from A to H. *(2 marks)*

(b) The airline Easygo introduces a competing route from G to H. Easygo advertises that travellers from A to H who include their new route can save £10 on Bryanair's lowest total fare. How much does Easygo charge for the route G to H? *(3 marks)*

(c) As an economy exercise, Bryanair decides to withdraw some of its routes. It wishes to run the cheapest set of routes which still keeps the eight cities connected.

(i) By applying Kruskal's algorithm, decide which routes should remain. State the order in which you have chosen the routes. *(5 marks)*

(ii) With this reduced set of routes, how much would it cost to fly from A to H **entirely** on Bryanair? *(2 marks)*

2 The statements **p** and **q** are:

p: it has snowed;
q: I shall ski.

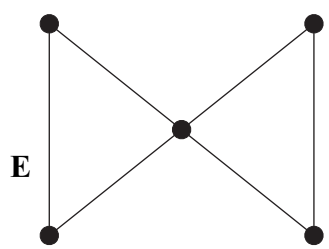
(a) Express the following sentences in terms of **p**, **q**, $\Rightarrow$ and $\sim$.

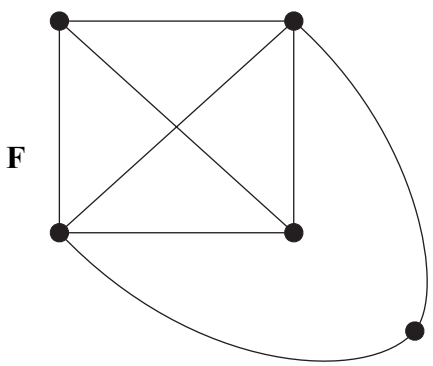
(i) If it has snowed then I shall ski. *(1 mark)*

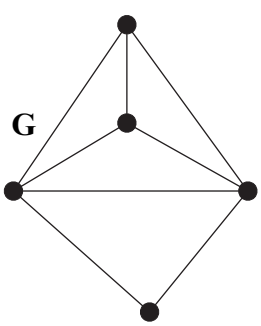
(ii) If it has not snowed then I shall not ski. *(1 mark)*

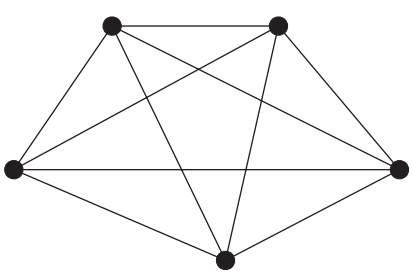
(b) Use truth tables to decide whether the sentences (i) and (ii) above are equivalent. *(4 marks)*

3 The four graphs **E**, **F**, **G** and **H** are illustrated below. They each have five vertices.









(a) State which, if any, of the four graphs are Eulerian. *(2 marks)*

(b) State which, if any, of the four graphs are Hamiltonian. *(2 marks)*

(c) State which, if any, of the four graphs are planar. *(2 marks)*

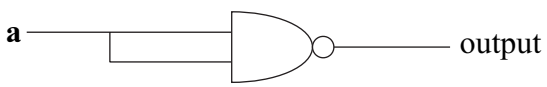
(d) State which pairs, if any, of the four graphs are isomorphic. *(2 marks)*

4 (a) The logical connective NAND $\uparrow$ is defined by

$$\mathbf{a} \uparrow \mathbf{b} = \sim(\mathbf{a} \wedge \mathbf{b})$$

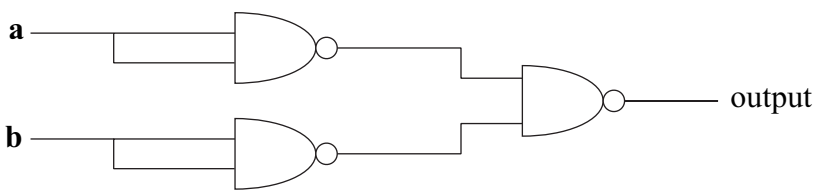
Give the truth table of NAND. *(2 marks)*

(b) The following circuit uses one NAND gate:



Show that the output is $\sim\mathbf{a}$. *(1 mark)*

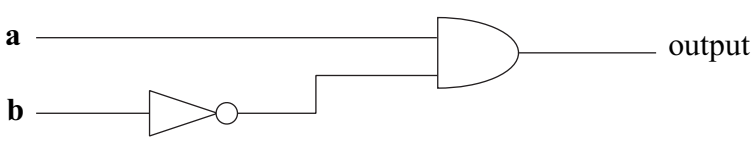
(c) The following circuit uses three NAND gates:



(i) Write down the output in terms of **a**, **b**, $\wedge$ and $\sim$. *(2 marks)*

(ii) Hence give a combinatorial circuit using a single gate which is equivalent to this circuit. *(2 marks)*

(d) The following circuit uses an AND gate and a NOT gate. Give a combinatorial circuit which is equivalent to it and uses only NAND gates.

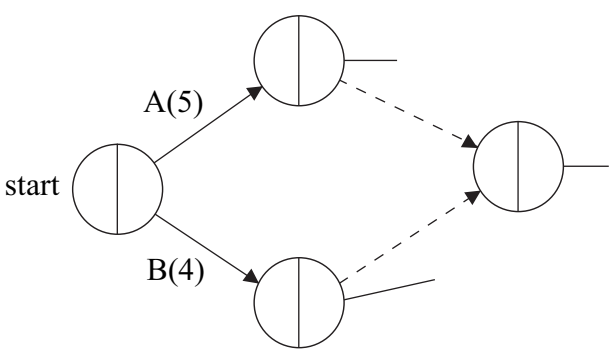


(3 marks)

5 A project consists of activities A – G as detailed in the table:

Activity	Duration (hours)	Cannot start until these activities are completed
A	5	–
B	4	–
C	4	A
D	7	B
E	7	A, B
F	3	C
G	5	D, E

This project is represented by an activity network, part of which is shown below:



(a) Copy the diagram and complete the activity network. *(3 marks)*

(b) Perform a forward and backward pass on your activity network in order to calculate all the early and late event times. *(4 marks)*

(c) State the minimum completion time for the project and list the critical activities. *(2 marks)*

(d) Each activity requires one worker.

(i) Show that the project cannot be completed in the minimum time with just two workers. *(2 marks)*

(ii) Find a schedule for the project which uses two workers and completes the project in just one hour more than the minimum completion time. *(3 marks)*

(e) It is discovered that an extra activity H is required. It cannot start until C is completed, and G cannot start until H is completed. Assuming that there is no restriction on the number of workers, calculate the maximum time which H can take without delaying the completion of the project. *(2 marks)*

6 [**Figure 2**, printed on the insert, is provided for use in answering part (b) of this question.]

Pisa Pizzas make two types of pizza base, the Romano and the Sardino.

The Romano requires 150 grams of flour, 50 grams of butter and 1 egg.
The Sardino requires 300 grams of flour, 20 grams of butter and 1 egg.
The available ingredients are $7\frac{1}{2}$ kg of flour, 1 kg of butter and 30 eggs.

Assume that Pisa Pizzas make x Romanos and y Sardinos.

(a) Show that the flour and butter requirements imply that x and y must satisfy

$$x + 2y \leqslant 50 \quad \text{and} \quad 5x + 2y \leqslant 100$$

and deduce an inequality from the egg requirements. *(3 marks)*

(b) On **Figure 2**, illustrate the region of those (x, y) which satisfy $x \geqslant 0$, $y \geqslant 0$ and the three inequalities in part (a). *(4 marks)*

(c) Pisa Pizzas make £1 profit on each Romano and £1.50 profit on each Sardino.

Find how many of each type they should make in order to maximise their profit. *(4 marks)*

(d) If instead Pisa Pizzas made £2 profit on each Romano and £1.50 profit on each Sardino, how many of each type should they make in order to maximise their profit? *(4 marks)*

7 (a) A simple graph has five vertices and their degrees are d, $d + 1$, $d + 1$, $d + 2$ and $d + 3$.

(i) By considering the sum of the degrees, show that d is odd. *(2 marks)*

(ii) Deduce that $d = 1$ and draw a graph with vertices having the given degrees. *(3 marks)*

(b) A simple graph has 10 vertices.

(i) Explain why the degree of each vertex must be in the set

$$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$ *(1 mark)*

(ii) Show that the degrees of the vertices cannot all be different. *(2 marks)*

END OF QUESTIONS

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Surname		Other Names	
Centre Number			Candidate Number				
Candidate Signature									

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Insert for use in Questions 1 and 6.

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Attach this sheet securely to your answer book.

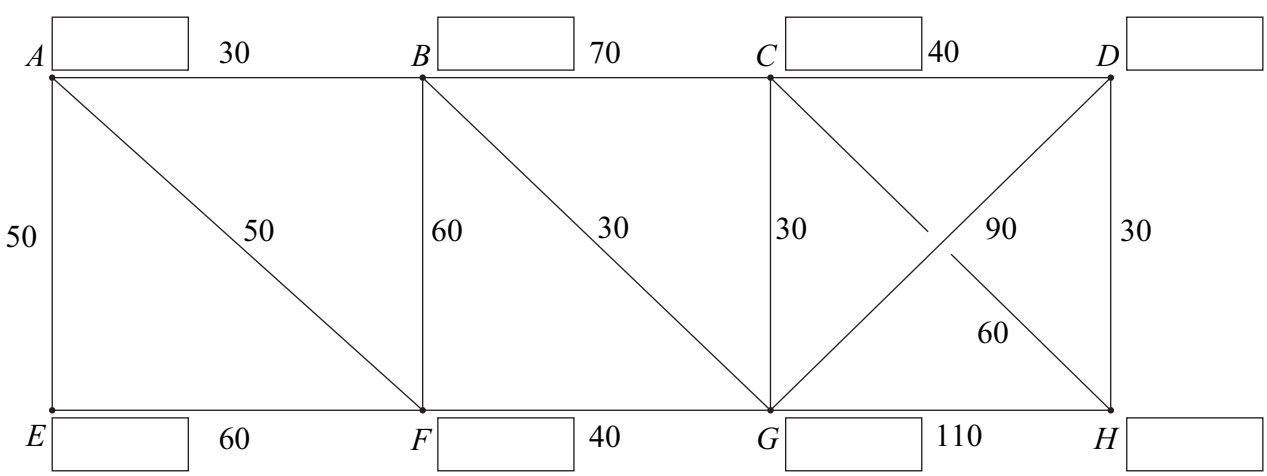

Figure 1

Turn over ▶

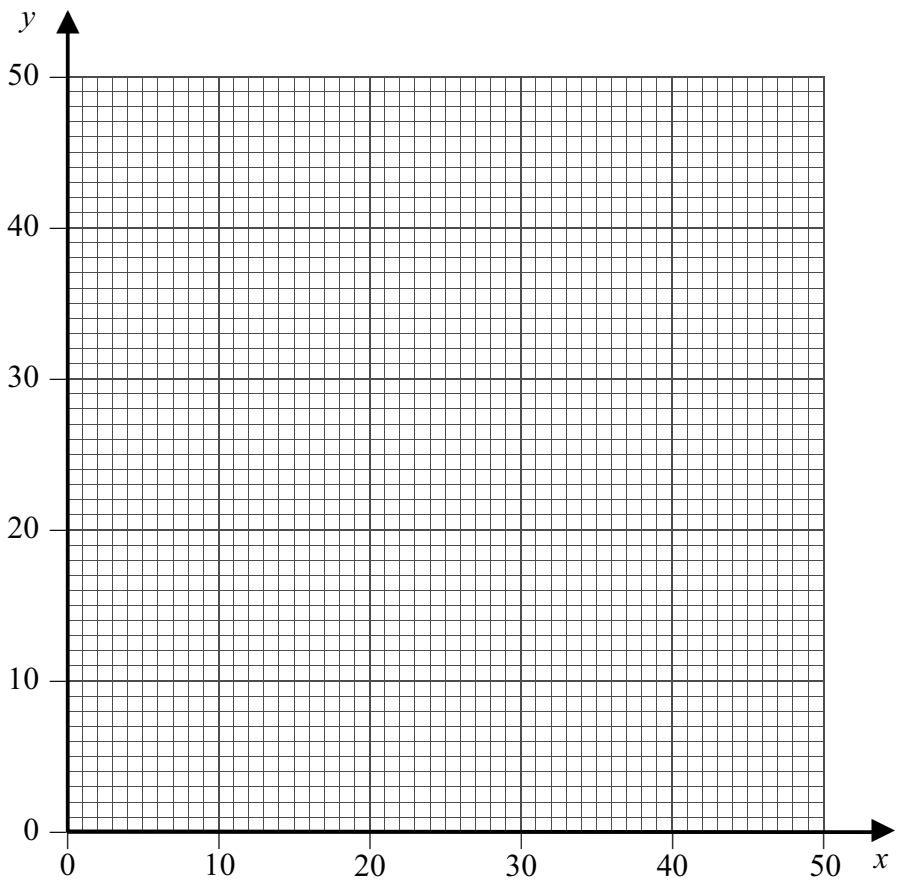

Figure 2