General Certificate of Education
June 2004
Advanced Subsidiary Examination

MATHEMATICS AND STATISTICS (SPECIFICATION B) Unit Discrete 1

MBD1

Friday 28 May 2004 Afternoon Session

In addition to this paper you will require:
- a 12-page answer book;
- the AQA booklet of formulae and statistical tables;
- an insert for use in Questions 1, 3 and 6 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 45 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBD1.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Fill in the boxes at the top of the insert. Make sure that you attach the insert to your answer book.

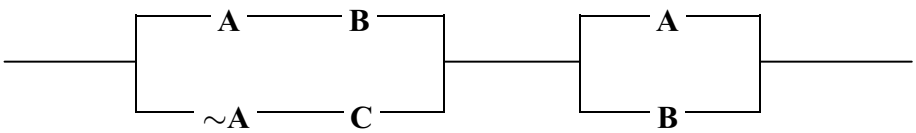
Information
- The maximum mark for this paper is 80.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 [**Figures 1** and **2**, printed on the insert, are provided for use in answering parts (a) and (b) of this question.]

Consider this switching circuit:

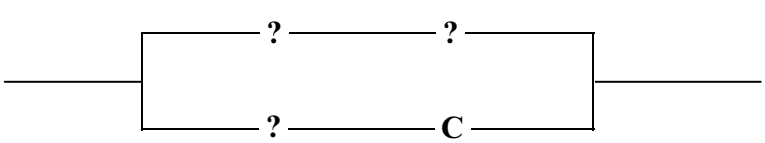


(a) On **Figure 1** complete the copy of the following table showing which values of **A**, **B** and **C** allow a flow through the circuit:

A	**B**	**C**	**Current flow?**
0	0	0	0
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

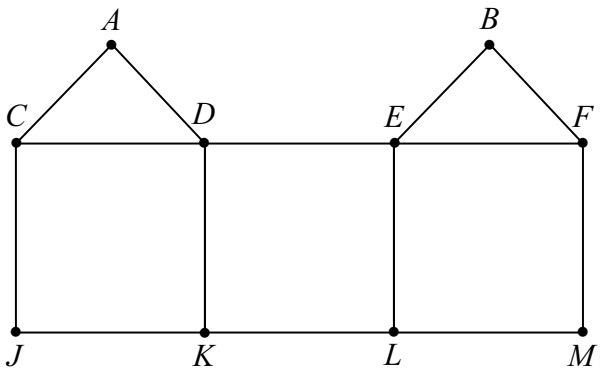
(4 marks)

(b) On **Figure 2** complete the following switching circuit, labelling each of the three remaining switches so that the circuit is equivalent to the one above.



(3 marks)

2 **G** is the graph illustrated.



(a) Find the Hamiltonian cycle in **G** which begins *CA* . . . . *(2 marks)*

(b) Is **G** Eulerian, semi-Eulerian or neither? Give a reason for your answer. *(2 marks)*

(c) One edge is removed from **G** to make a semi-Eulerian graph.

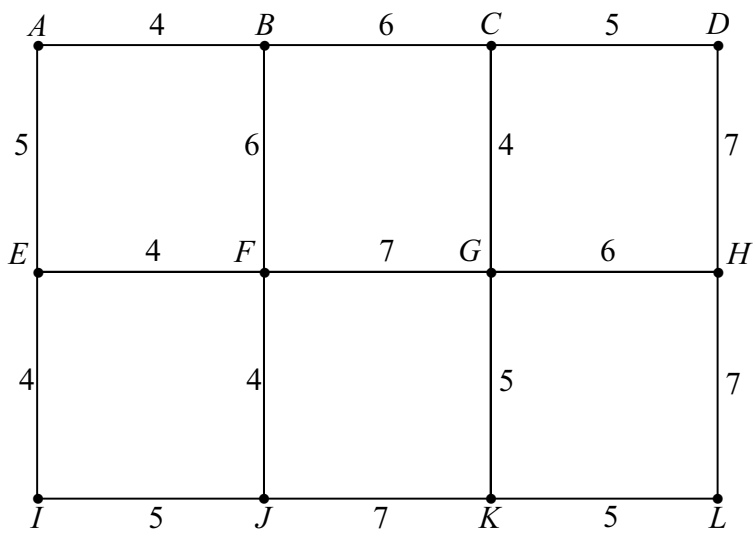
(i) State which edge is removed. *(1 mark)*

(ii) Explain why the resulting graph is not Hamiltonian. *(2 marks)*

TURN OVER FOR THE NEXT QUESTION

3 [**Figure 3**, printed on the insert, is provided for use in answering part (a) of this question.]

The following network represents twelve alpine huts *A*–*L* and the lengths, in kilometres, of any direct paths between them.



(a) Use Dijkstra's algorithm on **Figure 3** to find the shortest route from *A* to *L* on the paths. Show all your working at each vertex. *(6 marks)*

(b) In winter some paths have to be gritted.

(i) Use Kruskal's algorithm to find the minimum total length of paths which must be gritted so that it is possible to travel between any two of the huts on gritted paths. State the order in which you have chosen the paths. *(5 marks)*

(ii) Draw the tree representing the minimum connector which you found in part (b)(i). *(1 mark)*

(c) After a heavy snowfall the minimum paths are gritted, as in part (b), and the rest of the paths cannot be used.

(i) Calculate the distance from *J* to *L* on gritted paths. *(1 mark)*

(ii) Two friends staying in different huts walk on paths between these huts on the shortest possible route. After the snowfall the distance the friends have to walk is over four times what it was before. Find in which huts they are staying. *(2 marks)*

4 (a) The table shows the months of the year and the number of days in each month:

Month	Days
JANUARY	31
FEBRUARY	28 or 29
MARCH	31
APRIL	30
MAY	31
JUNE	30
JULY	31
AUGUST	31
SEPTEMBER	30
OCTOBER	31
NOVEMBER	30
DECEMBER	31

Let **j**, **y**, **t** and **u** be the following statements:

j: the month's first letter is J
y: the month's last letter is Y
t: the month has exactly 30 days
u: the month has exactly 31 days

Decide whether each of the following propositions is true, giving a reason in each case.

(i) $\mathbf{j} \Rightarrow \mathbf{u}$

(ii) $\mathbf{t} \Rightarrow \sim\mathbf{y}$

(iii) $(\mathbf{j} \wedge \mathbf{y}) \Rightarrow \mathbf{u}$ *(5 marks)*

(b) Let I and II be the following general propositions:

I: $\mathbf{p} \Rightarrow \mathbf{r}$ II: $(\mathbf{p} \wedge \mathbf{q}) \Rightarrow \mathbf{r}$

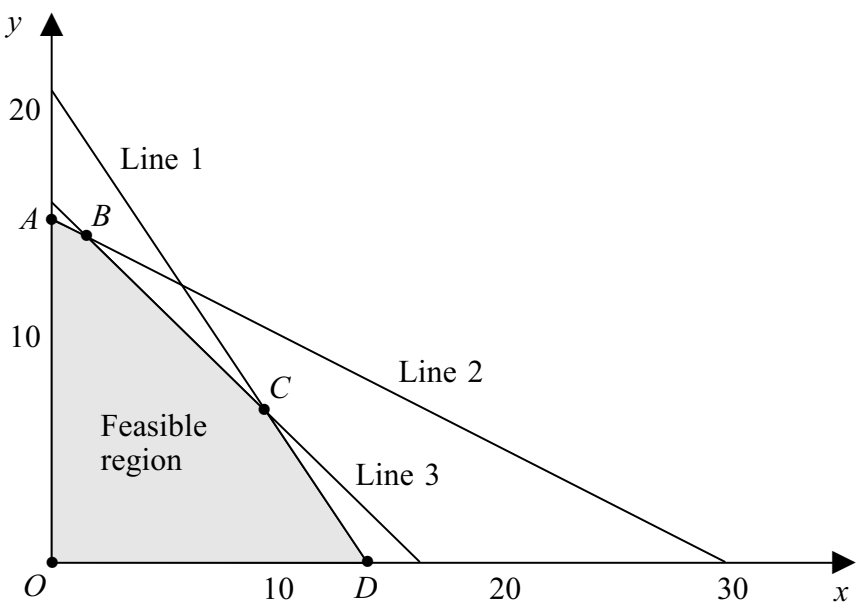
(i) Use truth tables to show that if I is true then so is II. *(5 marks)*

(ii) Using the context of the example in part (a), show how it is possible for statement II to be true when statement I is false. *(2 marks)*

5 A factory can produce two types of tractor, the Xtreme and the Yltra. The factory makes x of the Xtremes and y of the Yltras. In order to maximise the profit, P, the following linear programming problem has to be solved:

$$\begin{aligned} &\text{Maximise } P = 2x + 3y \\ &\text{subject to } x \geqslant 0,\ y \geqslant 0 \\ &x + 2y \leqslant 30 \\ &3x + 2y \leqslant 42 \\ &x + \ \ y \leqslant 16 \end{aligned}$$

The diagram below is a sketch in which the feasible region for this problem is shaded.



(a) State the equations of Lines 1, 2 and 3. *(2 marks)*

(b) Solve the simultaneous equations

$$\begin{aligned} 3x + 2y &= 42 \\ x + \ \ y &= 16 \end{aligned}$$

and hence state the coordinates of the vertex C. *(3 marks)*

(c) The coordinates of the other four vertices of the feasible region are

O (0, 0) A (0, 15) B (2, 14) D (14, 0).

Solve the above linear programming problem completely. *(4 marks)*

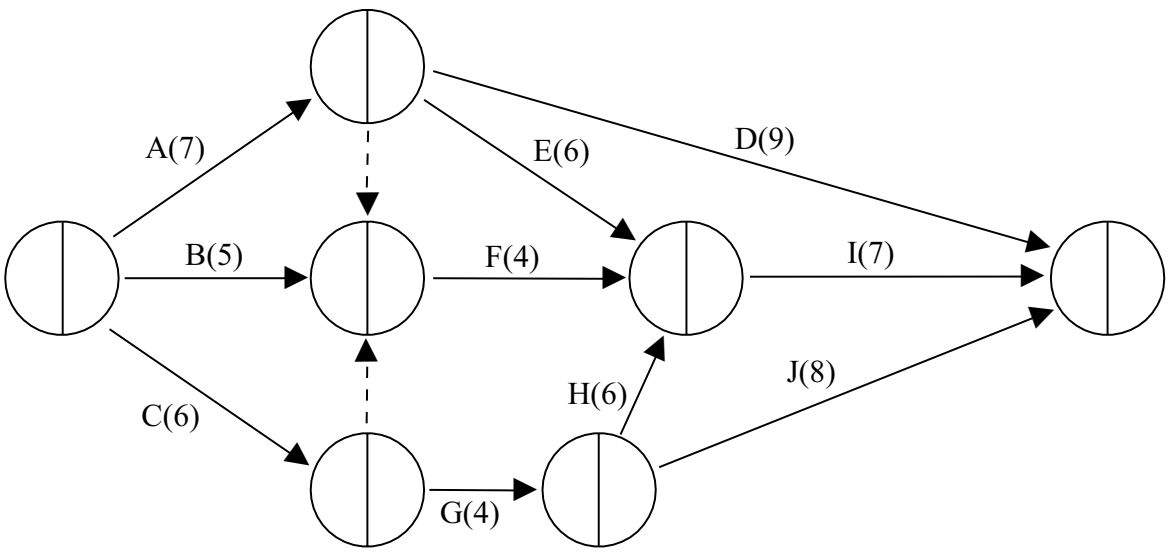
(d) The factory finds that it has an additional constraint that at most 20% of the tractors which it makes can be Yltras.

(i) Show that this extra condition is equivalent to $4y \leqslant x$. *(2 marks)*

(ii) Calculate how many of each type should now be made in order to maximise P. *(4 marks)*

6 [**Figures 4** and **5**, printed on the insert, are provided for use in answering parts (b) and (d) of this question.]

The activity network below represents a project involving activities A–J. The number on each arc is the duration of that activity in days.

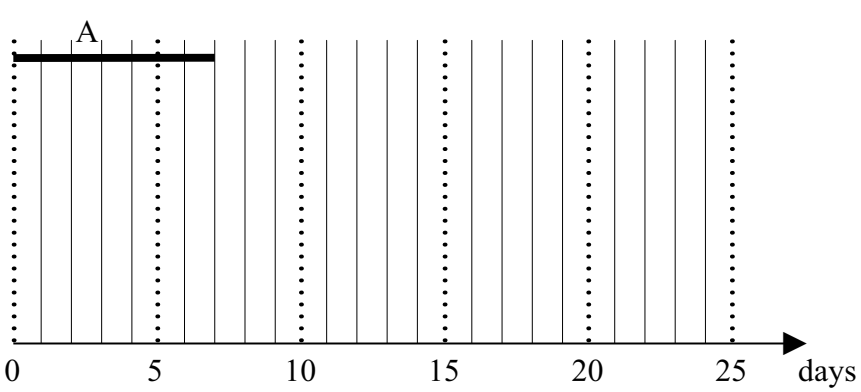


(a) State all the activities which must be completed before activity F can start. *(1 mark)*

(b) Perform a forward and backward pass on **Figure 4** in order to calculate all the early and late event times. *(5 marks)*

(c) State the minimum completion time for the project and list the critical activities. *(2 marks)*

(d) The following diagram begins to illustrate a schedule for the project in the minimum possible time with each activity starting as early as possible:



On **Figure 5** complete this schedule. *(4 marks)*

(e) Each activity requires one worker. Show how to adapt your schedule in part (d) so that the project can be completed in the minimum time with just three workers. *(2 marks)*

7 A simple graph has six vertices and their degrees are d, d, d, d, $d - 3$ and $2d - 3$.

(a) (i) By considering the sum of the degrees show that d is even. *(2 marks)*

(ii) Deduce that d must be 4. *(2 marks)*

(b) Draw a graph which satisfies the given description. *(2 marks)*

(c) State whether your graph in part (b) is planar or not. Give a reason for your answer. *(2 marks)*

(d) Decide whether all the graphs which satisfy the given description are isomorphic, giving a reason. *(2 marks)*

END OF QUESTIONS

Surname		Other Names	
Centre Number						Candidate Number				
Candidate Signature										

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MATHEMATICS AND STATISTICS (SPECIFICATION B)
Unit Discrete 1

MBD1

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Insert for use in Questions 1, 3 and 6.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

TURN OVER FOR FIGURE 1

A	B	C	Current flow?
0	0	0	0
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Figure 1

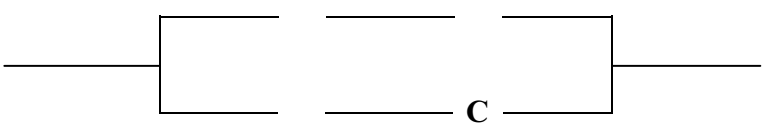

Figure 2

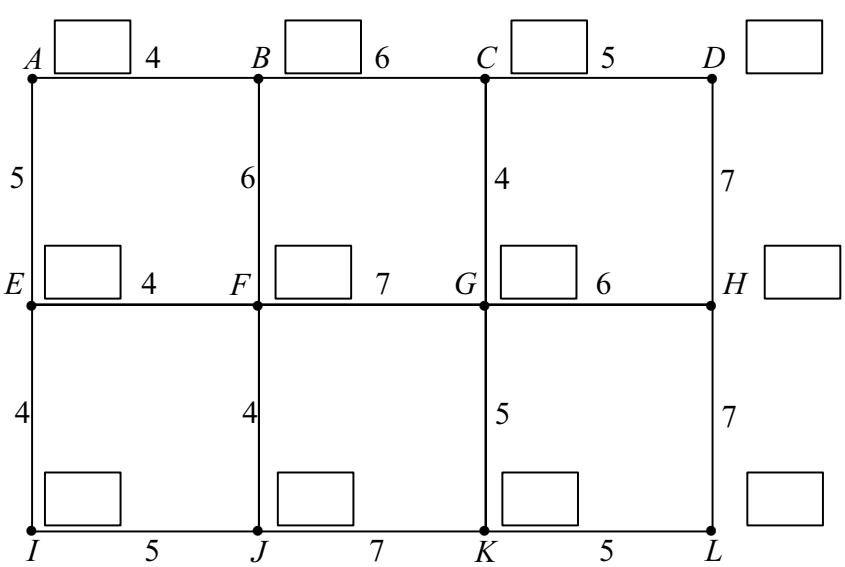

Figure 3

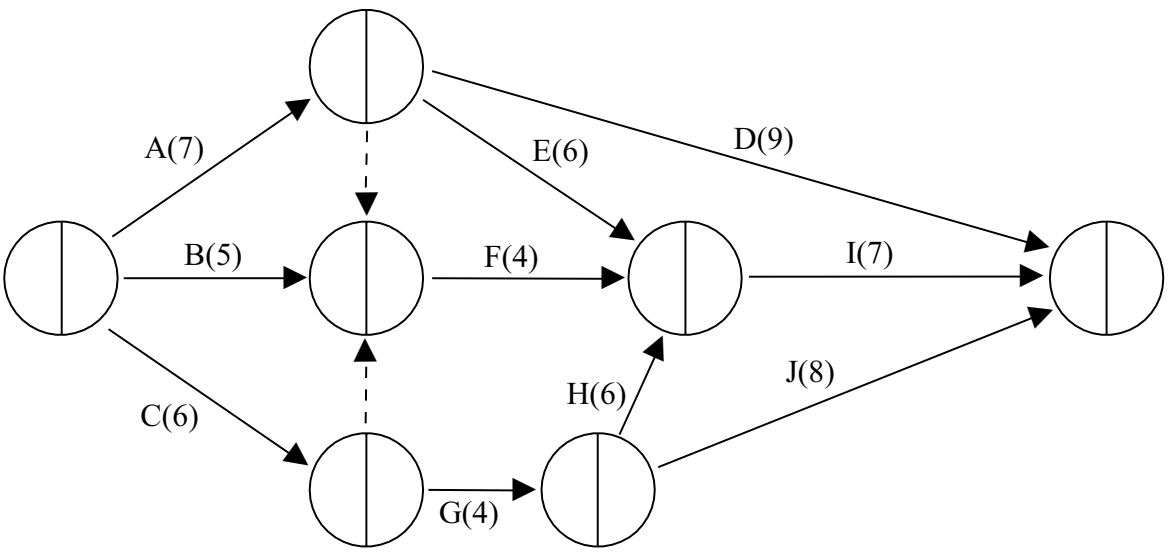

Figure 4

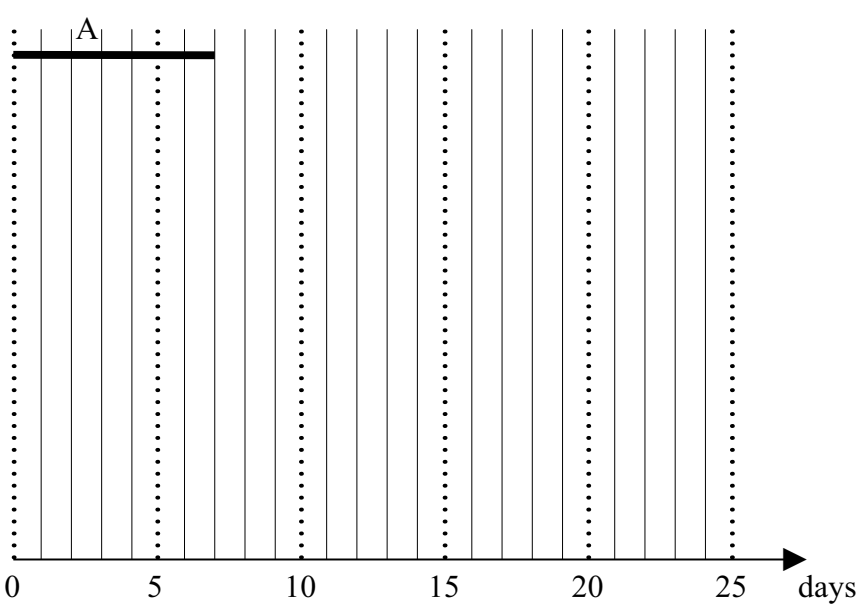

Figure 5

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