General Certificate of Education
January 2005
Advanced Subsidiary Examination

MATHEMATICS AND STATISTICS (SPECIFICATION B) Unit Discrete 1

MBD1

Wednesday 12 January 2005 Afternoon Session

In addition to this paper you will require:
- a 12-page answer book;
- an insert for use in Questions 1, 2, 3, 5 and 6 (enclosed);
- a ruler;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 45 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBD1.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Fill in the boxes at the top of the insert. Make sure that you attach the insert to your answer book.

Information
- The maximum mark for this paper is 80.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 [**Figure 1**, printed on the insert, is provided for use in answering part (a) of this question.]

A group of seven friends, P, Q, R, S, T, U and V, keep in touch by phone. The minimum cost, in pence, of a phone call between any pair of them is shown in the following table.

	P	***Q***	***R***	***S***	***T***	***U***	***V***
P	–	30	50	45	30	35	60
Q	30	–	40	40	80	70	55
R	50	40	–	45	80	70	70
S	45	40	45	–	35	60	60
T	30	80	80	35	–	30	60
U	35	70	70	60	30	–	50
V	60	55	70	60	60	50	–

Person P wishes to pass on a piece of news to all the other friends, either by a direct phone call or by the message being passed on from friend to friend.

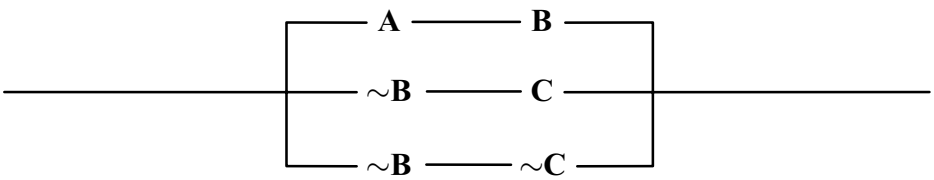
(a) By applying Prim's algorithm to the matrix in **Figure 1**, find the minimum cost of notifying all the friends of the news. *(5 marks)*

(b) Draw a graph representing the minimum connector which you found in part (a). *(2 marks)*

(c) Person Q is on holiday and so a message can be left on her answerphone but she cannot be used to pass the message on. In these new circumstances, what is the minimum cost of notifying all the friends of the news? *(2 marks)*

2 [**Figure 2**, printed on the insert, is provided for use in answering part (a) of this question.]

Consider this switching circuit:



(a) On **Figure 2** complete the copy of the following table showing which values of **A**, **B** and **C** allow a flow through the circuit.

A	B	C	Current flow?
0	0	0	1
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

(4 marks)

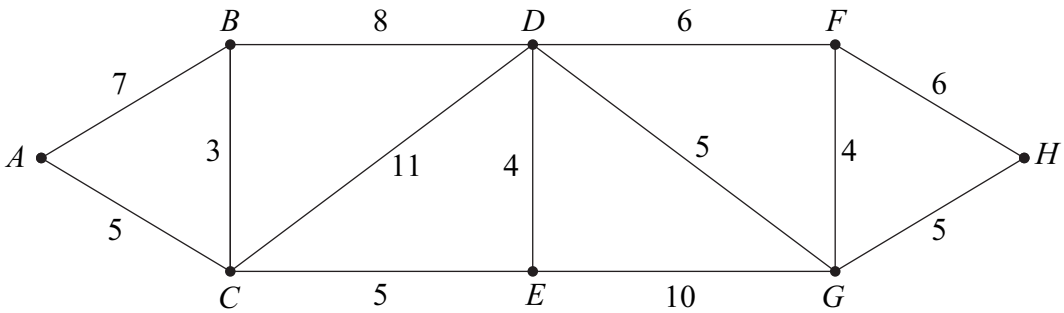
(b) Give a switching circuit with just two switches which is equivalent to the one above.

(3 marks)

TURN OVER FOR THE NEXT QUESTION

3 [**Figure 3**, printed on the insert, is provided for use in answering part (a) of this question.]

The network shows the distances, in miles, on main roads between eight towns A–H.



(a) Use Dijkstra's algorithm on **Figure 3** to find the shortest route from A to H on these roads. Show all your working. *(6 marks)*

(b) Cars travel on the roads at 30 miles per hour.

(i) Calculate how many minutes the journey of shortest length from A to H will take. *(1 mark)*

(ii) A new major road is opened from A to D along which cars will be able to travel at 60 miles per hour. This new road will enable the journey time from A to H to be reduced to 36 minutes. Calculate the length of the new road from A to D. *(3 marks)*

4 Let **p**, **q** and **r** be the statements:

p: I am over 17;
q: I have passed my driving test;
r: I can buy a car.

(a) Express the following sentences in terms of **p**, **q**, **r**, $\sim$, $\wedge$ and $\Rightarrow$.

(i) If I am over 17 and I have passed my driving test then I can buy a car.

(ii) I can only buy a car if I have passed my driving test.

(iii) I cannot buy a car if I am not over 17. *(6 marks)*

(b) The three sentences in part (a) can actually be combined into one sentence symbolised by

$$\mathbf{r} \Leftrightarrow (\mathbf{p} \wedge \mathbf{q})$$

Put this sentence into words. *(2 marks)*

(c) Use Boolean algebra to simplify the expression

$$(\mathbf{p} \vee \sim\mathbf{q}) \wedge \mathbf{q}$$ *(3 marks)*

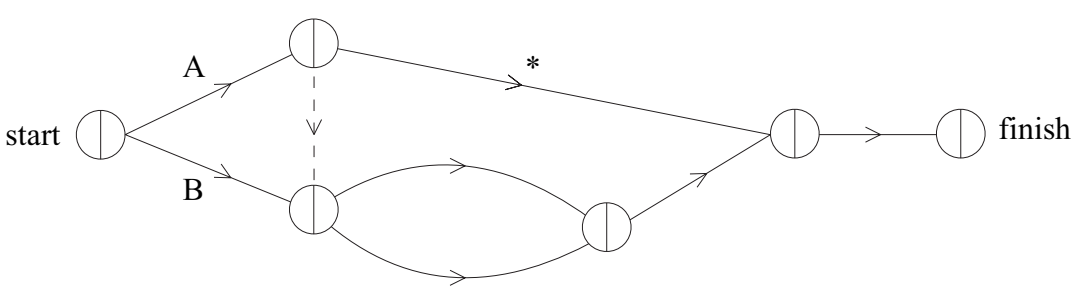
TURN OVER FOR THE NEXT QUESTION

5 [**Figure 4**, printed on the insert, is provided for use in answering this question.]

To assemble a computer desk from a kit requires various activities. These are shown in the following table, together with how many minutes each activity takes and the order in which some of them must be done.

Activity	**Duration** (minutes)	**Cannot be started until these are finished**
A: assemble the drawers	40	–
B: fix the legs to the desk-top	20	–
C: attach the drawers	20	A
D: attach the printer shelf	20	A, B
E: attach the scanner shelf	10	A, B
F: attach the sockets	20	D, E
G: wire-up all the fittings	30	C, F

The following network is an activity-on-arc network for this project.



(a) By considering which activity must be completed before the activity marked * can start, deduce which activity is represented by the arc *. *(1 mark)*

(b) On **Figure 4** label each of the remaining arcs, apart from the dummy arc, with its appropriate letter to complete the activity network. *(2 marks)*

(c) Perform forward and backward passes in order to find the early and late event times for your network. *(4 marks)*

(d) State the minimum number of minutes in which the project can be completed and list the critical activities. *(2 marks)*

(e) State which activities have an independent float. *(2 marks)*

(f) Each activity requires one worker. Give a schedule for the project in the minimum completion time which uses a first worker for the entire time and a second worker for just 50 consecutive minutes. *(3 marks)*

6 [**Figure 5**, printed on the insert, is provided for use in answering part (b) of this question.]

A jeweller works for up to seven days a week. He makes necklaces and bracelets. He makes one necklace on each working day and on some working days he also makes a bracelet. He makes a total of at most 10 items each week.

Assume that the jeweller makes x necklaces and y bracelets in a week.

(a) Explain why x and y must satisfy

$$0 \leqslant x \leqslant 7, \quad 0 \leqslant y \leqslant 7 \quad \text{and} \quad y \leqslant x$$

and write down another inequality which x and y must satisfy. *(3 marks)*

(b) On **Figure 5** illustrate the region representing those (x, y) which satisfy all the inequalities in part (a). *(4 marks)*

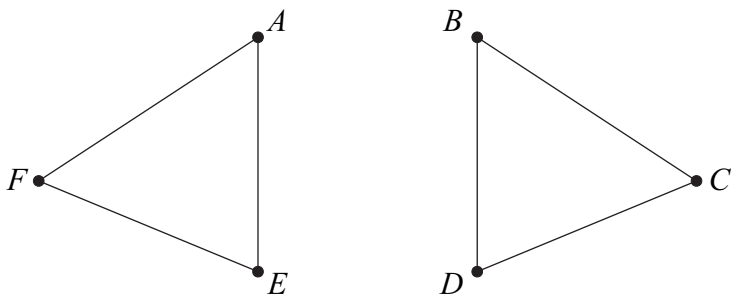
(c) The jeweller charges £20 for a necklace and £10 for a bracelet.

(i) Find how many necklaces and bracelets he should make each week in order to maximise his income. *(4 marks)*

(ii) The jeweller wants to increase the cost of the bracelets but leave the cost of the necklaces unchanged. He wants to choose the new price so that when maximising his income he need only work five days a week. Calculate the minimum that he must charge for each bracelet. *(4 marks)*

TURN OVER FOR THE NEXT QUESTION

7 The graph **G** has six vertices $A-F$ and is illustrated below.



(a) What is the minimum number of edges which must be added to **G** in order to make the graph connected? *(1 mark)*

(b) What is the minimum number of edges which must be added to **G** in order to make the graph Hamiltonian? State a suitable set of edges. *(2 marks)*

(c) (i) State the minimum number of edges which must be added to **G** in order to create a semi-Eulerian graph. Draw such a graph and write down a trail of this graph which uses all its edges. *(3 marks)*

(ii) Show that by adding two edges between the same pair of vertices you can create an Eulerian graph. *(1 mark)*

(iii) State the minimum number of edges which must be added to **G** in order to create a **simple** graph which is Eulerian. Draw such a graph. *(3 marks)*

(d) Seven edges are added to **G** to create another simple graph on the same six vertices.

(i) Show that the new graph consists of K_6 with two edges removed. *(2 marks)*

(ii) Deduce that the new graph is not planar. *(2 marks)*

END OF QUESTIONS

Surname		Other Names	
Centre Number			Candidate Number								
Candidate Signature											

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	P	***Q***	***R***	***S***	***T***	***U***	***V***
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Q	30	–	40	40	80	70	55
R	50	40	–	45	80	70	70
S	45	40	45	–	35	60	60
T	30	80	80	35	–	30	60
U	35	70	70	60	30	–	50
V	60	55	70	60	60	50	–

Figure 1

A	B	C	Current flow?
0	0	0	1
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Figure 2

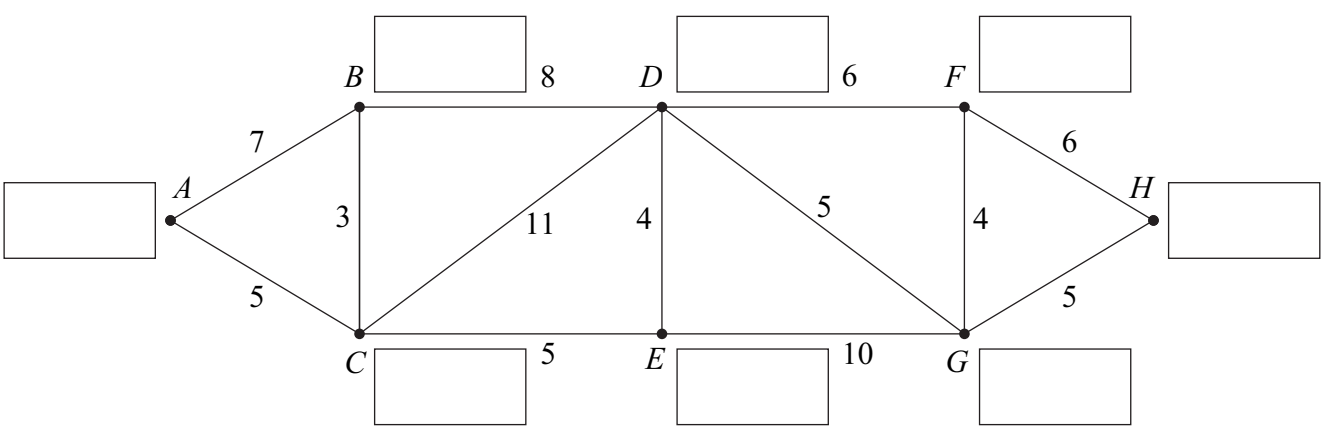

Figure 3

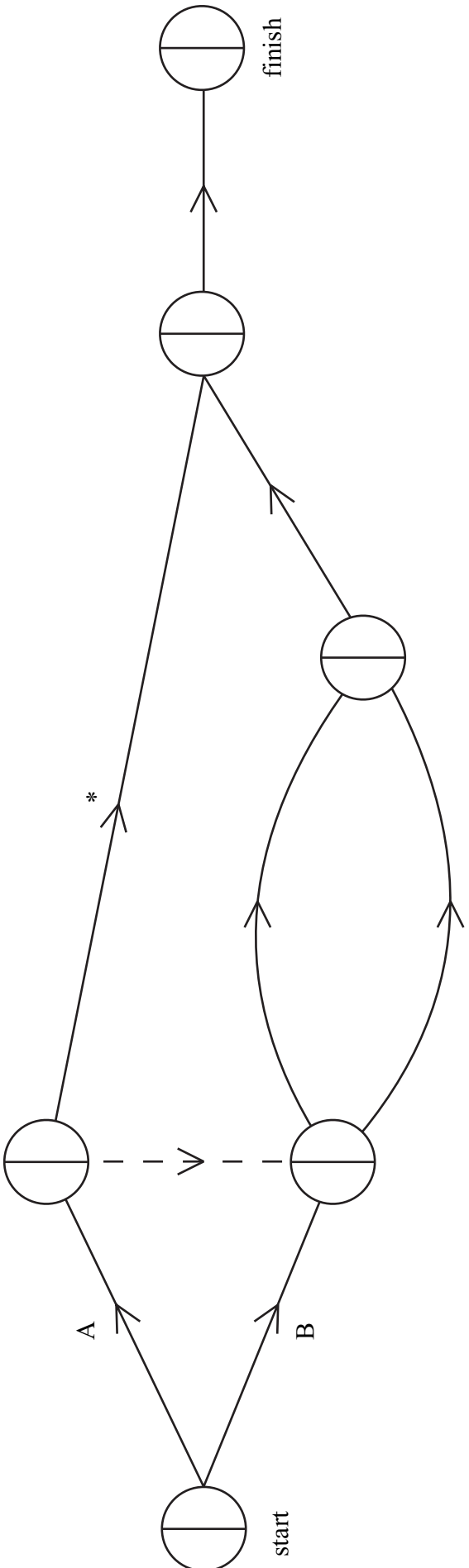

Figure 4

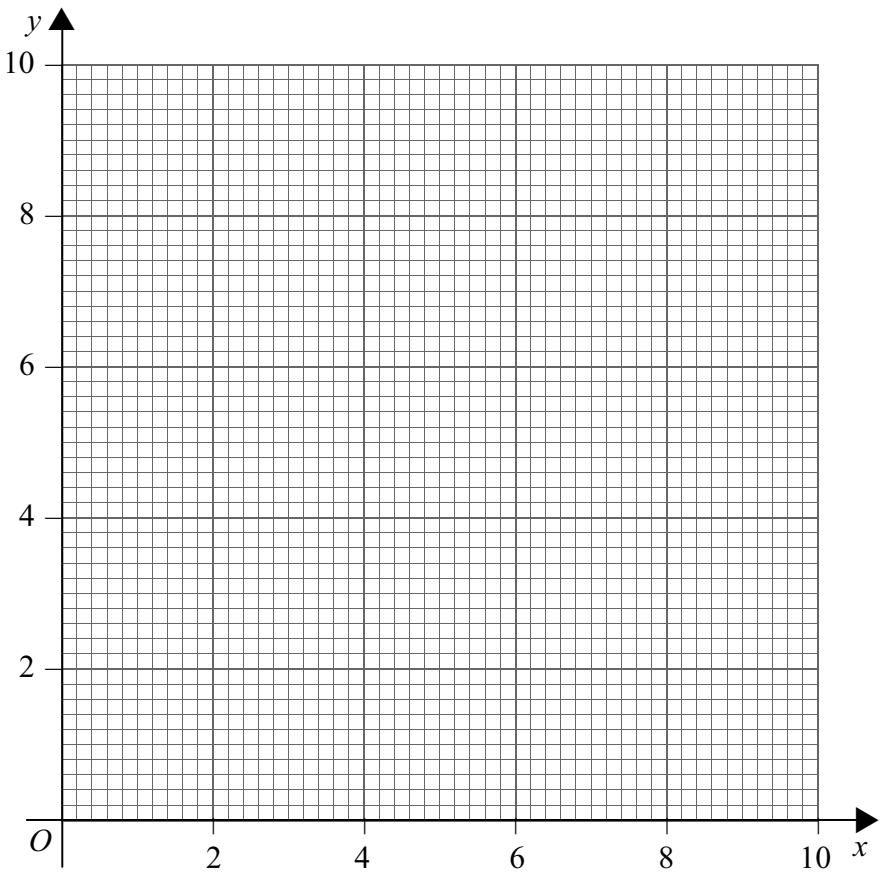

Figure 5