

## General Certificate of Education

# Mathematics 6300

## *Specification A*

*MAS4/W Statistics 4*

# Mark Scheme

*2005 examination – June series*

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.



## Key to Mark Scheme

<b>M</b>	mark is for	method
<b>m</b>	mark is dependent on one or more M marks and is for	method
<b>A</b>	mark is dependent on M or m marks and is for	accuracy
<b>B</b>	mark is independent of M or m marks and is for	accuracy
<b>E</b>	mark is for	explanation
<b>✓ or ft or F</b>		follow through from previous incorrect result
<b>CAO</b>		correct answer only
<b>AWFW</b>		anything which falls within
<b>AWRT</b>		anything which rounds to
<b>AG</b>		answer given
<b>SC</b>		special case
<b>OE</b>		or equivalent
<b>A2,1</b>		2 or 1 (or 0) accuracy marks
<b>-x EE</b>		deduct $x$ marks for each error
<b>NMS</b>		no method shown
<b>PI</b>		possibly implied
<b>SCA</b>		substantially correct approach
<b>c</b>		candidate
<b>sf</b>		significant figure(s)
<b>dp</b>		decimal place(s)

## Abbreviations used in Marking

<b>MC – <math>x</math></b>	deducted $x$ marks for mis-copy
<b>MR – <math>x</math></b>	deducted $x$ marks for mis-read
<b>ISW</b>	ignored subsequent working
<b>BOD</b>	given benefit of doubt
<b>WR</b>	work replaced by candidate
<b>FB</b>	formulae book

## Application of Mark Scheme

<b>No method shown:</b>	
Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise
<b>More than one method / choice of solution:</b>	
2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only
<b>Crossed out work</b>	do not mark unless it has not been replaced
<b>Alternative solution</b> using a correct or partially correct method	award method and accuracy marks as appropriate

**MAS4/W**

Q	Solution	Marks	Total	Comments																																				
<b>1(a)</b>	$S_{xx} = 49277 - \frac{689^2}{10} = 1804.9$	B1	5																																					
	$S_{yy} = 58679 - \frac{759^2}{10} = 1070.9$	B1																																						
$S_{xy} = 53052 - \frac{689 \times 759}{10} = 756.9$	B1																																							
$r = \frac{756.9}{\sqrt{1804.9 \times 1070.9}} = 0.544$	M1 A1																																							
<b>(b)</b> Low positive correlation; possibly not significant	E1	1																																						
<b>Total</b>			<b>6</b>																																					
<b>2</b>	$\frac{82}{144} \pm 1.96 \sqrt{\frac{\frac{41}{72} \times \frac{31}{72}}{144}}$	B1 M1 A1	4	z value																																				
	(0.489, 0.650)	A1		AWRT																																				
<b>Total</b>			<b>4</b>																																					
<b>3(a)(i)</b>	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th>Rank for</th> <th>Rank against</th> <th>d<sup>2</sup></th> </tr> </thead> <tbody> <tr><td>1</td><td>10</td><td>81</td></tr> <tr><td>3</td><td>5</td><td>4</td></tr> <tr><td>6</td><td>6.5</td><td>0.25</td></tr> <tr><td>2</td><td>9</td><td>49</td></tr> <tr><td>4.5</td><td>4</td><td>0.25</td></tr> <tr><td>4.5</td><td>8</td><td>12.25</td></tr> <tr><td>7.5</td><td>6.5</td><td>1</td></tr> <tr><td>7.5</td><td>3</td><td>20.25</td></tr> <tr><td>10</td><td>2</td><td>64</td></tr> <tr><td>9</td><td>1</td><td>64</td></tr> <tr><td></td><td></td><td><b>296</b></td></tr> </tbody> </table>	Rank for	Rank against	d <sup>2</sup>	1	10	81	3	5	4	6	6.5	0.25	2	9	49	4.5	4	0.25	4.5	8	12.25	7.5	6.5	1	7.5	3	20.25	10	2	64	9	1	64			<b>296</b>	M1 A1 A1 m1 A1	7	ranking for against  $\sum d^2$  AWRT  accept $r$ on ranks $r = -0.810$
	Rank for	Rank against	d <sup>2</sup>																																					
	1	10	81																																					
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	7.5	3	20.25																																					
	10	2	64																																					
	9	1	64																																					
		<b>296</b>																																						
$r_s = 1 - \frac{6 \times 296}{10 \times 99} = -0.794$	m1 A1																																							
<b>(ii)</b> Information is lost by ranking	E1	1																																						
<b>(b)</b>	$H_0: \rho_s = 0 \quad H_1: \rho_s < 0$	B1	4	both allow 0.7333 allow 0.794 > 0.7333																																				
	c.v.(1%) = -0.7333	B1																																						
	-0.794 < -0.7333	m1																																						
	Reject $H_0$ So implying $\rho_s < 0$	A1																																						
<b>Total</b>			<b>12</b>																																					

## MAS4/W (cont)

Q	Solution	Marks	Total	Comments
4(a)	$H_0: p = 0.2 \quad H_1: p > 0.2$	B1		both
	$X \sim \text{Bin}(50, 0.2)$	M1		allow $X \sim N(10, 8)$ or $p \sim N(0.2, 0.0032)$
	$P(X \geq 15) = 1 - 0.9393 = 0.0607$	A1		$Z_{\text{calc}} = 1.59$ (= 1.768 without continuity correction)
	$0.0607 > 0.05 \Rightarrow$ retain $H_0$ No evidence of improvement at the 5% level	A1✓	4	$Z_{\text{crit}} = 1.6449$
	(b) $P(X \leq 17) = 0.9937 \Rightarrow$ 18 is the least value to show improvement at 1% level	M1 A1	2	$10 + 2.3263\sqrt{8}$ $= 16.579 \Rightarrow 17$
<b>Total</b>			<b>6</b>	
5(a)	$y = cd^x$ $\Rightarrow \ln y = \ln c + x \ln d$ $\Rightarrow y = mx + c$	M1 A1	2	
	(b) $S_{xy} = 61.107 - \frac{26.492 \times 22}{10} = 2.8246$ $S_{xx} = 72 - \frac{22^2}{10} = 23.6$ $b = \frac{2.8246}{23.6} = 0.11968\dots$ $\ln \bar{y} = 2.6492, \quad \bar{x} = 2.2$ $a = 2.6492 - 0.11968\dots \times 2.2$ $= 2.38588\dots$ $\ln y = 2.39 + 0.120x$	M1 A1 B1 M1 A1	5	<b>full data:</b> 0.11967... both 2.38595... accept 0.12
(c)	$\ln c = 2.385\dots \Rightarrow c = 10.9$ $\ln d = 0.11968 \Rightarrow d = 1.13$	M1 A1 A1	3	anti logging AWRT
(d)	$y = cd^x$ $= 14.7$	M1 A1	2	14.66 accept AWRT 14.7 or 14.8 CSO ( $11 \times 1.1^{2.5} = 14.6$ is M1A0)
<b>Total</b>			<b>12</b>	

MAS4/W (cont)

Q	Solution	Marks	Total	Comments
6	$H_0: P_S - P_A = 0 \quad H_1: P_S - P_A \neq 0$	B1		both
	$P_S = \frac{91}{250} = 0.364 \quad P_A = \frac{73}{250} = 0.292$	B1		both (may be implied by working)
	$Z_{\text{calc}} = \frac{0.072 - 0}{\sqrt{\frac{0.364 \times 0.636}{250} + \frac{0.292 \times 0.708}{250}}}$	M1		variance
		M1		accept pooling
	$= 1.7196$	A1		$\hat{p} = 0.328$
	$Z_{\text{crit}} = \pm 1.96$ $\Rightarrow$ Retain $H_0$ Support is the same in both faculties at 5% level	B1  A1✓	7	$Z_{\text{calc}} = 1.7146$
<b>Total</b>			<b>7</b>	
7(a)	$E\left(\frac{X-b}{a}\right) = \frac{1}{a}E(X-b) = \frac{1}{a}E(X) - \frac{b}{a}$	M1		
	$= \frac{1}{a}(a\lambda + b) - \frac{b}{a} = \lambda$	M1 A1	3	
(b)(i)	$E(X) = \frac{2}{9} \int_{\lambda}^{\lambda+3} (\lambda+3)x - x^2 dx$	M1		LNR
	$= \frac{2}{9} \left[ (\lambda+3) \frac{x^2}{2} - \frac{x^3}{3} \right]_{\lambda}^{\lambda+3}$	A1		
	$= \frac{2}{9} \left\{ \frac{[\lambda+3]^3}{2} - \frac{[\lambda+3]^3}{3} - \left[ \frac{(\lambda+3)\lambda^2}{2} - \frac{\lambda^3}{3} \right] \right\}$	A1		inserting limits
	$= \frac{2}{9} \left\{ \frac{(\lambda+3)^3}{6} - \frac{3\lambda^3 + 9\lambda^2 - 2\lambda^3}{6} \right\}$	m1		common denominator
	$= \frac{2}{9} \left\{ \frac{\lambda^3 + 9\lambda^2 + 27\lambda + 27 - \lambda^3 - 9\lambda^2}{6} \right\}$	A1		
	$= \lambda + 1$ Unbiased estimator is $X - 1$	A1 A1	7	or $\bar{X} - 1$
(ii)	$\bar{X} = \frac{30}{5} = 6$	B1		
	$\hat{\lambda} = 6 - 1 = 5$	M1 A1✓	3	use of any value for $X$ ft on $\bar{X}$ only
<b>Total</b>			<b>13</b>	
<b>TOTAL</b>			<b>60</b>	