

General Certificate of Education
January 2005
Advanced Level Examination



MATHEMATICS (SPECIFICATION A)
Unit Statistics 3

MAS3

Monday 31 January 2005 Morning Session

In addition to this paper you will require:

- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 20 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAS3.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information

- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

- 1 Vikram grows tomato plants of the same variety in two different composts, A and B. The weights of tomatoes from plants grown in each compost are normally distributed with standard deviation 5 grams.

Vikram suspects that plants grown in compost A produce heavier tomatoes on average than those grown in compost B. To investigate this suspicion he takes a random sample of 10 tomatoes from plants grown in compost A and records the weight, x grams, of each tomato. He also takes a random sample of 8 tomatoes from plants grown in compost B and records the weight, y grams, of each tomato.

Vikram finds that the difference between the means of the two samples, $\bar{x} - \bar{y}$, is 3.75.

Use a test, at the 5% level of significance, to investigate Vikram's suspicion. (8 marks)

- 2 A beekeeper sells jars of honey which are labelled, "Total weight: 300 grams". She takes a random sample of 10 filled jars and records the weight, x grams, of each filled jar.

Her results are summarised below, with \bar{x} denoting the sample mean.

$$\sum x = 3030 \qquad \sum (x - \bar{x})^2 = 148$$

- (a) Calculate unbiased estimates of the mean, μ , and the variance, σ^2 , of the weight, X grams, of a jar of honey. (2 marks)
- (b) Assuming that the weights of jars of honey are normally distributed, construct a 98% confidence interval for:
- (i) the mean, μ ; (5 marks)
- (ii) the standard deviation, σ . (5 marks)
- (c) A customer claims that the jar of honey he bought weighed only 270 grams.

Use your confidence intervals, calculated in part (b), to comment on his claim.

(3 marks)

3 The random variable T has an exponential distribution with mean 5.

(a) (i) Specify fully the probability density function, $f(t)$, for T . (2 marks)

(ii) Show that, for $t \geq 0$, T has distribution function

$$F(t) = 1 - e^{-0.2t}. \quad (2 \text{ marks})$$

(iii) Find the probability that a randomly chosen value of T lies between 2 and 8. (3 marks)

(b) A roadside snack bar opens at 7.30 am each day. There is a probability of 0.15 that at least one customer is already waiting when the snack bar opens. Otherwise the time, T minutes, until the first customer arrives can be modelled by an exponential distribution with mean 5.

(i) On a particular day, there are no customers waiting when the snack bar opens.

Find the probability that no customers have arrived by 7.40 am. (2 marks)

(ii) For a randomly selected day, find the probability that the first customer arrives before 7.35 am. (3 marks)

4 A therapist works with chronically ill patients. He believes that self-hypnosis can help such patients in the relief of pain.

The therapist takes a random sample of 12 patients and teaches them techniques of self-hypnosis. Two weeks later he asks them what change there has been in their levels of pain. The results are as follows.

Patient	1	2	3	4	5	6	7	8	9	10	11	12
Less pain	✓	✓		✓	✓	✓	✓		✓	✓		✓
No change											✓	
More pain			✓					✓				

(a) (i) Name the test that you would use to investigate the therapist's belief. (1 mark)

(ii) Give a reason for your choice of test. (1 mark)

(b) Using the 5% level of significance, carry out the test that you named in part (a)(i) to investigate the therapist's belief. (7 marks)

- 5 Katy reads two newspapers, the Courier and the Journal. She always completes the crossword in each newspaper. Her completion times, X minutes for a Courier crossword and Y minutes for a Journal crossword, are normally distributed random variables.

Katy records her completion time, x minutes, for each of a random sample of 7 Courier crosswords, and her completion time, y minutes, for each of a random sample of 9 Journal crosswords. She calculates unbiased estimates of the means, μ_X and μ_Y , and the variances, σ_X^2 and σ_Y^2 , of the variables X and Y , with the following results.

	Unbiased estimate of mean	Unbiased estimate of variance
Courier crosswords	$\bar{x} = 19.4$	$s_X^2 = 9.88$
Journal crosswords	$\bar{y} = 15.9$	$s_Y^2 = 6.24$

- (a) Use a test at the 10% significance level to show that it is reasonable to believe that $\sigma_X = \sigma_Y$. (6 marks)
- (b) (i) Assuming that $\sigma_X = \sigma_Y$, construct a 95% confidence interval for the difference in means, $\mu_X - \mu_Y$. (8 marks)
- (ii) Katy believes that she is quicker, on average, at solving Journal crosswords than at solving Courier crosswords.

Use your confidence interval to comment on her belief. (2 marks)

END OF QUESTIONS