AQA
ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark scheme January 2004

GCE

Mathematics A

Unit MAS3

Key to mark scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m mark and is for	accuracy
B	mark is independent of M or m marks and is for	method and accuracy
E	mark is for	explanation
√ or ft or F		follow through from previous incorrect result
CAO		correct answer only
AWFW		anything which falls within
AWRT		anything which rounds to
AG		answer given
SC		special case
OE		or equivalent
A2,1		2 or 1 (or 0) accuracy marks
– *x* EE		Deduct *x* marks for each error
NMS		No method shown
PI		Perhaps implied
c		Candidate

Abbreviations used in marking

MC – *x*	deducted *x* marks for miscopy
MR – *x*	deducted *x* marks for misread
ISW	ignored subsequent working
BOD	gave benefit of doubt
WR	work replaced by candidate

Application of mark scheme

Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise

Award method and accuracy marks as appropriate to an alternative solution using a correct method or partially correct method.

Q	Solution	Marks	Total	Comments
1	$s_x{}^2 = \frac{1}{9}\left[569.45 - \frac{(75.43)^2}{10}\right]$	M1		Use of
	$= 0.0535 \text{ mm}^2$	A1	2	AWFW 0.053 to 0.054
(b)	Assume diameters are normally distributed.	B1		
	$H_0 : \sigma_x = 0.15$ or $\sigma_x{}^2 = 0.0225$			
	$H_1 : \sigma_x = 0.15$ or $\sigma_x{}^2 > 0.0225$	B1		Both
	Significance level = 0.05			
	Degrees of freedom $v = 10 - 1 = 9$	B1		CAO
	Critical value of $c = 16.919$	B1		CAO
	Sample value of $\chi^2 = \frac{(n-1)s^2}{\sigma^2}$			
	$= \frac{9 \times 0.0535}{0.0225}$	M1		ft on $s_x{}^2$
	$= 21.4$	A1√		ft on s_x^2; AWFW 21.2 to 21.6
	$\chi^2{}_{calc} > \chi^2{}_{crit}$			
	Reject H_0. Evidence at 5% level of an increase in standard deviation (or variance)	A1√	7	ft on $\chi^2{}_{\text{calc}}$ and $\chi^2{}_{\text{crit}}$
(c)	Change in mean would cause more large bolts or more small bolts but not both.	E1		
	Increase in standard deviation means wider spread of diameters so consistent with more at both extremes	E1	2	E2 if clear that variability is to be tested and why
	Total		**11**	

Q	Solution	Marks	Total	Comments
2	Sample ratio = $\frac{s_x^2}{s_y^2}$			
	$= \frac{2.4049}{0.5372}$	M1		
	$= 4.477$	A1		AWFW 4.47 to 4.48
	Degrees of freedom $v_1 = v_2 = 11$	B1		CAO both
	95% confidence interval so $p = 0.975$			
	Critical value $F_{11,11} = 3.474$	B1		CAO
	$\frac{1}{F} \le \frac{\sigma_x^2 / s_x^2}{\sigma_y^2 / s_y^2} \le F$	M1		Use of
	$\frac{1}{3.474} \le \frac{\sigma_x^2 / \sigma_y^2}{4.477} \le 3.474$	A1√		ft on ratio and F value
	Confidence interval is (1.29, 15.6)	A1	7	(AWRT 1.29, AWFW 15.5 to 15.6)
(b)	Lower confidence limit > 1	E1√		ft on CI
	Journey time is more variable from home to school than returning.	E1√	2	ft consistent with CI
	Total		**9**	
3 (a)	H_0 : P(prefer luxury blend) = 0.5	B1		
	H_1 : P(prefer luxury blend) > 0.5			
	Ignoring zero differences, sample size $n = 9$	B1		CAO; may be implied
	X = Number who prefer luxury blend.			
	Under H_0 $X \sim B(9, 0.5)$	B1√		ft on n: may be implied
	Actual value of $X = 7$ (or 2)	B1		CAO
	$P(X \ge 7) = P(x \le 2)$	M1		
	= 0.0898	A1√		ft on n and X
	0.0898 < 10% so reject H_0			
	Evidence supports the claim that the luxury blend is preferred.	A1√	7	ft on probability
(b)(i)	Makes use of more information – takes into account size as well as direction of differences.	E1	1	
(ii)	Scores are subjective so differences cannot be reliably ranked.	E1	1	
	Total		**9**	

Q	Solution	Marks	Total	Comments
4 (a)	$F(t)=\int_0^t \frac{1}{2}e^{\frac{-x}{2}}\,dx$	M1		Integration and limits
	$=\left[-e^{-\frac{x}{2}}\right]_0^t$	A1		Correct integration
	$=1-e^{-\frac{t}{2}}$		2	Printed answer
(b)(i)	$P(T\le 3)\;=F(3)$			
	$=1-e^{-1.5}$	M1		or by integration
	$=0.777$	A1	2	AWFW 0.776 to 0.777
(ii)	We require $P(T>1)$	M1		Could be implied
	$=1-F(1)$	m1		
	$=e^{-0.5}=0.607$	A1	3	AWFW 0.606 to 0.607
(iii)	We require $P(T\le 3\mid T>1)$	M1		Could be implied
	$=\frac{P(1\le T\le 3)}{P(T>1)}$			
	$=\frac{F(3)-F(1)}{1-F(1)}$	M1		or by integration
	$=\frac{(1-e^{-1.5})-(1-e^{-0.5})}{e^{-1.5}}$	A1√		or uses answers to (i) and (ii) ft on previous answers
	$=0.632$	A1	4	AWFW 0.630 to 0.634 B1 for just F(3) – F(1)
(c)	$X=3+T$	M1		Could be implied
	Mean = 3 + 2 = 5	A1		CAO
	Standard deviation = 2	A1	3	CAO
	Total		**14**	

Q	Solution	Marks	Total	Comments
5 (a)	$H_0: \quad \mu_x = 1.7$ $H_1: \quad \mu_x \neq 1.7$	B1		Both
	$\alpha = 0.10$			
	Degrees of freedom $v = 8 - 1 = 7$	B1		CAO
	Critical values of $t = \pm 1.895$	B1		AWFW 1.89 to 1.90
	Sample statistic $t = \dfrac{\bar{x} - \mu_x}{\sqrt{\dfrac{s_x^2}{n}}}$	M1		Use of
	$= \dfrac{17.51 - 17}{\sqrt{\dfrac{1.273}{8}}}$	A1		All terms correct
	$= 1.28$	A1		AWFW 1.27 to 1.28
	Sample t lies within -1.895 to $+1.895$ so reasonable to accept that $\mu_x = 17$	A1√	7	ft on t and critical value
(b)(i)	Pooled estimate of σ^2			
	$= \dfrac{(7 \times 1.273) + (8 \times 1.719)}{8 + 9 - 2}$	M1		
	$= 1.511$	A1		AWRT 1.51
	$\bar{y} - \bar{x} = 14.30$	B1		CAO
	Degrees of freedom $v = 15$	B1		CAO
	95% interval $\Rightarrow p = 0.975$			
	Critical value of $t = 2.131$	B1		AWFW 2.13 to 2.14
	Confidence limits for $\mu_y - \mu_x$ are			
	$14.30 \pm 2.131 \times \sqrt{1.511} \times \sqrt{\dfrac{1}{8} + \dfrac{1}{9}}$	M1 A1√		ft on t and σ^2
	95% confidence interval is (13.0, 15.6)	A1	8	AWFW (13.0 to 13.1, 15.5 to 15.6)
(b)(ii)	75% of 17 = 12.75			
	Or CI for % increase is (76.65, 91.59)	B1		
	75% lies below lower confidence limit so the claim is supported.	E1√	2	ft on CI
	Total		**17**	
	Total		**60**	