General Certificate of Education

Mathematics 6300
Specification A

MAS2/W Statistics 2

Mark Scheme

2005 examination - June series

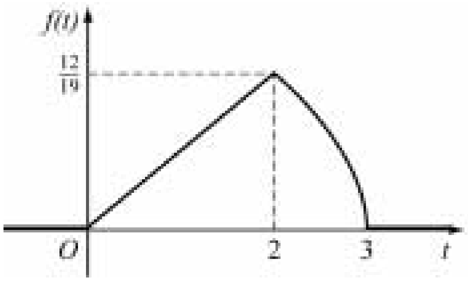
Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

MAS2/W

Q	Solution	Marks	Total	Comments
1	$X \sim \text{Po}(50)$			
	$X \sim \text{N}(50, 50)$	B1		CAO
	$\text{P}(X \geq 60) = \text{P}\left(Z > \frac{59.5 - 50}{\sqrt{50}}\right)$	M1		use of continuity correction
		M1		(standardisation)
	$= \text{P}(Z > 1.3435)$			
	$= 1 - \Phi(1.34)$	m1		dependent on (standardisation)
	$= 1 - 0.90988$			
	$= 0.0901$	A1	5	AWFW 0.0895 to 0.0902
	Total		**5**	
2(a)	$Y \sim \text{Geo}\ (0.8)$	B1	1	
(b)	$\text{P}(Y = 5) = (0.2)^4(0.8)$	M1		
	$= 0.00128$	A1	2	
(c)	$\text{E}(Y) = \frac{1}{p} = \frac{1}{0.8} = 1.25$	B1		CAO
	$\text{Var}(Y) = \frac{q}{p^2} = \frac{0.2}{0.64} = \frac{5}{16}$			
	$= 0.3125$	B1	2	CAO
	Total		**5**	

MAS2/W (cont)

Q	Solution	Marks	Total	Comments
3(a)		B2	2	B1 for straight line on [0, 2] B1 for curve on [2, 3]
(b)(i)	$\mathrm{P}(T<0.5)=\frac{1}{2}\times\frac{1}{2}\times\frac{3}{19}=\frac{3}{76}$	M1		or by integration: $\int_0^{0.5}\frac{6t}{19}\,\mathrm{d}t=\left[\frac{3t^2}{19}\right]_0^{0.5}$
	$=0.0395$	A1	2	$=\frac{3}{76}=0.0395$ (AG)
(ii)	$Y\sim$ number of times Suneil has to wait for less than 30 seconds			
	$Y\sim\mathrm{B}\,(50,\,0.0395)$	B1		
	Distributional approximation: $\mu=50\times0.0395=1.975$ $\sigma^2=1.975\times0.9605=1.90$ $\therefore Y\approx\mathrm{Po}(1.975)$	B1		AWFW 1.97 to 1.98
	$\mathrm{P}(Y<4)=$ $\mathrm{e}^{-1.97}\left(1+1.97+\frac{1.97^2}{2!}+\frac{1.97^3}{3!}\right)$	M1A1		
	$=0.8616$	A1	5	AWFW 0.860 to 0.862
(c)	$\mathrm{E}(T)=\frac{6}{19}\int_0^2 t^2\,\mathrm{d}t+\frac{6}{19}\int_2^3 t^2(3-t)\,\mathrm{d}t$	M1		
	$=\left[\frac{2t^3}{19}\right]_0^2+\left[\frac{6t^3}{19}-\frac{3t^4}{38}\right]_2^3$	A1A1		
	$=\frac{16}{19}+\frac{33}{38}$	M1		
	$=\frac{65}{38}$ $=1.71$	A1	5	CAO
	Total		**14**	

MAS2/W (cont)

<table>
<tr><th>Q</th><th>Solution</th><th>Marks</th><th>Total</th><th>Comments</th></tr>
<tr><td>4</td><td>$H_o : X \sim N(160, 64)$

<table><tr><th>X</th><th>z</th><th>p</th></tr><tr><td>$X < 150$</td><td>-1.25</td><td>0.1056</td></tr><tr><td>$150 < X < 158$</td><td>$(-1.25, -0.25)$</td><td>0.2957</td></tr><tr><td>$158 < X < 162$</td><td>$(-0.25, 0.25)$</td><td>0.1974</td></tr><tr><td>$162 < X < 170$</td><td>$(0.25, 1.25)$</td><td>0.2957</td></tr><tr><td>$X > 170$</td><td>1.25</td><td>0.1056</td></tr><tr><td></td><td></td><td>$\sum p = 1$</td></tr></table>
<table><tr><th>O_i</th><th>E_i</th><th>$(O_i - E_i)^2 / E_i$</th></tr><tr><td>11</td><td>21.12</td><td>4.8492</td></tr><tr><td>67</td><td>59.14</td><td>1.0446</td></tr><tr><td>31</td><td>39.48</td><td>1.8214</td></tr><tr><td>64</td><td>59.14</td><td>0.3994</td></tr><tr><td>27</td><td>21.12</td><td>1.6370</td></tr><tr><td>$\sum O_i = 200$</td><td>$\sum E_i = 200$</td><td>$\sum = 9.7517$</td></tr></table>
$\nu = 5 - 1 = 4$

$\chi^2_{5\%}(4) = 9.488$
$\therefore$ reject H_o
The evidence suggests that $N(160, 64)$ is not a suitable model</td><td>M1

A1
A1

M1
M1

A1

M1

M1

A1

B1

B1√

A1√</td><td>

12</td><td>use of $z = \frac{x - \mu}{\sigma}$

$z = \pm 1.25$
$z = \pm 0.25$

$p = 0.1056$
$p = 0.2957$
$p = 0.1974$ and $\sum p = 1$

$E_i = 200 \times p_i$

use of $\sum \frac{(O_i - E_i)^2}{E_i}$

AWFW 9.6 to 9.8

AWRT 9.49

ft on χ^2 and critical value</td></tr>
<tr><td></td><td>**Total**</td><td></td><td>**12**</td><td></td></tr>
</table>

MAS2/W (cont)

Q	Solution	Marks	Total	Comments
5(a)(i)	$(Y-X) \sim \text{N}(2, 6.25)$	B1 B1	 2	for Normal and $\mu = 2$ for 6.25
(ii)	$\text{P}(Y-X<0) = \text{P}\left(Z < \frac{0-2}{2.5}\right)$	M1		$z = \frac{0 - \text{their } \mu}{\text{their } r}$
	$= \text{P}(Z < -0.80)$	A1√		
	$= 1 - \Phi(0.80)$ $= 1 - 0.78814$			
	$= 0.21186$ $= 0.212$	 A1	 3	 AWRT 0.212
(b)	$B = X_1 + X_2 + X_3 + X_4 \sim \text{N}(64, 9)$ and $G = Y_1 + Y_2 + Y_3 + Y_4 \sim \text{N}(72, 16)$	B1		
	$\therefore\ (B-G) \sim \text{N}(-8, 25)$	M1A1		$(G-B) \sim N(8, 25)$
	$\text{P}(\lvert B-G \rvert < 5)$ $= \text{P}\left(\frac{-5-(-8)}{5} < Z < \frac{5-(-8)}{5}\right)$	M1		
	$= \text{P}(0.6 < Z < 2.6)$ $= \Phi(2.6) - \Phi(0.6)$	A1		
	$= 0.99534 - 0.72575$			
	$= 0.26959$	A1		
	$\therefore\ \text{P}(\lvert B-G \rvert > 5) = 0.730$	A1	7	AWRT 0.730 **Alternative:** $\text{P}[(B-G) < -5] + \text{P}[(B-G) > 5] =$ $\Phi(0.6) + [1 - \Phi(2.6)]$ = 0.7257 + 0.0047 = 0.73041 = 0.730
	Total		**12**	

MAS2/W (cont)

Q	Solution	Marks	Total	Comments
6(a)	$H_0 : \mu = 65$	B1		
	$H_1 : \mu > 65$	B1	2	
(b)	$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right) = N(65, 0.64)$	B1		for 0.64
		B1	2	for Normal and 65
(c)	$P(\text{Type I error}) =$			
	$P(H_0 \text{ rejected when } H_0 \text{ true})$			
	$= P(\bar{X} > 66.4 \text{ when } \mu = 65)$			
	$= P\left(Z > \frac{66.4 - 65}{0.8}\right)$	M1		
	$= P(Z > 1.75)$	m1		area change
	$= 1 - 0.95994$			
	$= 0.04006$	A1		AWRT 0.040
	$\therefore$ significance level of test $\approx$ 4%	A1√	4	ft on Type I error
(d)	$P(\text{Type II error}) =$			
	$P(H_0 \text{ accepted when } H_0 \text{ false})$			
	$P(\bar{X} < 66.4 \text{ when } \mu = 67)$			
	$= P\left(Z < \frac{66.4 - 67}{0.8}\right)$	M1		
	$= P(Z < -0.75)$	A1		
	$= 1 - \Phi(0.75)$			
	$= 1 - 0.77337$	m1		
	$= 0.22663$			
	$= 0.227$	A1	4	AWRT 0.23
	Total		**12**	
	TOTAL		**60**	