GCE 2005

January Series

ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark Scheme

Mathematics A

(MAS2)

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

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Key to Mark Scheme

M .... mark is for ....method
m................................... mark is dependent on one or more M marks and is for ....method
A .... mark is dependent on M or m marks and is for ....accuracy
B .... mark is independent of M or m marks and is for .... method and accuracy
E .... mark is for .... explanation
√ or ft or F ....follow through from previous incorrect result
CAO .... correct answer only
AWFW.. anything which falls within
AWRT.. anything which rounds to
AG ....answer given
SC ....special case
OE .... or equivalent
A2,1 ....2 or 1 (or 0) accuracy marks
-*x* EE .... deduct *x* marks for each error
NMS ....no method shown
PI .... possibly implied
SCA .... substantially correct approach
c ....candidate
SF .... significant figure(s)
DP...decimal place(s)

Abbreviations used in Marking

MC – *x* .... deducted *x* marks for mis-copy
MR – *x* .... deducted *x* marks for mis-read
ISW ....ignored subsequent working
BOD ....given benefit of doubt
WR .... work replaced by candidate
FB...formulae booklet

Application of Mark Scheme

No method shown:
Correct answer without working .... mark as in scheme
Incorrect answer without working.. zero marks unless specified otherwise

More than one method/choice of solution:

2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only
Crossed out work	do not mark unless it has not been replaced
Alternative solution using a correct or partially correct method	award method and accuracy marks as appropriate

MAS2

Q	Solution	Marks	Total	Comments
1(a)(i)	$S \sim B(12, 0.09)$	B1	1	
(ii)	$\text{P}(S \geq 4) = 1 - \text{P}(S \leq 3)$	M1		AWRT
	$= 1 - 0.9820$			
	$= 0.018$	A1	2	
(b)	Let X = number of boxes which contain 4. or more eggs made with white chocolate			
	Then: $X \sim \text{B}(300, 0.018)$	B1		CAO
	$np = 300 \times 0.018 = 5.4$			
	$npq = 5.4 \times 0.982 \approx 5.3$			
	$\Rightarrow$ use a Poisson approximation	M1		
	$X \approx \text{P}_\text{o}(5.4)$	A1		CAO
	$\text{P}(X \geq 3) = 1 - \text{P}(X \leq 2)$	M1		$e^{-5.4}\left(1 + 5.4 + \frac{5.4^2}{2}\right)$
	$= 1 - \{\text{P}(X=0) + \text{P}(X=1) + \text{P}(X=2)\}$			$= e^{-5.4} \times 20.98$
	$= 1 - 0.0948$			$= 1 - 0.09476$
	$= 0.905$	A1	5	AWRT $= 0.905$
	Total		**8**	

MAS2 (cont)

Q	Solution	Marks	Total	Comments
2(a)	$P(2 \le X \le 3) = F(3) - F(2)$	M1		
	$= \frac{27}{64} - \frac{1}{8}$			
	$= \frac{19}{64} = 0.297$	A1	2	CAO/AWRT
(b)	$f(x) = \begin{cases} \frac{3x^2}{64} & 0 \le x \le 4 \end{cases}$	M1A1	2	
(c)(i)	$E(X) = \int_0^4 x.f(x)\ dx$			
	$= \int_0^4 \frac{3x^3}{64} dx$	M1		
	$= \left. \frac{3x^4}{256} \right\|_0^4$	A1√		
	$= 3$	A1	3	CAO
(ii)	$\text{Var}(X) = \int x^2 f(x) dx - [E(X)]^2$			
	$= \int_0^4 \frac{3x^4}{64} dx - (3)^2$	M1		
	$= \left. \frac{3x^5}{320} \right\|_0^4 - 9$	A1√		on their f(*x*) and μ^2
	$= 9\frac{3}{5} - 9$			
	$= \frac{3}{5}$ or 0.6	A1	3	CAO
	Total		**10**	

MAS2 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$X \sim \text{Geo}(0.1)$	B1	1	
(b)	$\text{P}(X=4)=(0.9)^3\times(0.1)$	M1		
	$=0.0729$	A1	2	AWRT
(c)	$\text{P}(X \le n)=1-(0.9)^n$	M1		
	$\therefore 1-(0.9)^n \ge 0.95$			
	$\Rightarrow (0.9)^n \le 0.05$	M1		
	$n=28 \quad \Rightarrow \quad (0.9)^{28}=0.0523$	M1		AWRT
	$n=29 \quad \Rightarrow \quad (0.9)^{29}=0.0471$	A1		CAO
	The minimum number of days = 29		4	
	Total		**7**	

MAS2 (cont)

Q	Solution	Marks	Total	Comments
4	H_0 : no association between their gender and type of holiday preferred	B1		(for at least (H_0))
	H_1 : there is an association between their gender and type of holiday			
	Totals:			
	118, 98, 216 90, 114, 204 208, 212, 420	B1		CAO
	Expected frequencies:			
	$\frac{216\times208}{420}=106.97$			
	$\frac{216\times212}{420}=109.03$	M1		
	$\frac{204\times208}{420}=101.03$	A1		
	$\frac{204\times212}{420}=102.97$			
	$v=1$	B1		
	$\sum\frac{\left[\lvert O_i-E_i\rvert-0.5\right]^2}{E_i}$	M1		Yates' correction
	$=1.036+1.017+1.097+1.077$			
	$=4.227$:	A1		AWFW 4.20 to 4.24
	$\chi^2_{5\%}(1)=3.841$	B1		(4.20 if E_i rounded to neareast integer)
	$4.23>3.841$ $\therefore$ reject H_o	A1√		(Only if H_0 stated)
	Evidence at the 5% level of an association between the gender of single people and the type of holiday that they prefer to take.	E1√	10	
	Total		**10**	

MAS2 (cont)

Q	Solution	Marks	Total	Comments
5(a)(i)	$\mathrm{E}(X-Y)=120-90=30$	B1	1	CAO
(ii)	$\mathrm{Var}(X-Y)=36+13=49$	B1	1	CAO
(b)	$\mathrm{P}(X-Y>40.5)$			
	$=\mathrm{P}\left(Z>\dfrac{40.5-30}{7}\right)$	M1		CAO
	$=\mathrm{P}(Z>1.5)$	A1√		On their μ and r
	$=1-\Phi(1.5)$			
	$=1-0.93319$	m1		
	$=0.06681$	A1	4	AWRT 0.0668
(c)(i)	$\mathrm{E}(T)=\mathrm{E}(X_1)+\mathrm{E}(X_2)+2\mathrm{E}(Y)$			
	$=120+120+180$			
	$=420$	B1		CAO
	$\mathrm{Var}(T)=\mathrm{Var}(X_1)+\mathrm{Var}(X_2)$			
	$+4\mathrm{Var}(Y)$	M1		
	$=36+36+52$			
	$=124$	A1	3	CAO
(ii)	$T\sim N(420,124)$			
	$\mathrm{P}(T<400)=\mathrm{P}\left(Z<\dfrac{400-420}{\sqrt{124}}\right)$	M1		
	$=P(Z<-1.796)$	A1√		
	$=1-0.96407$	M1		
	$=0.03593$	A1	4	AWFW 0.0355 to 0.0365
	Total		**13**	

MAS2 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$H_o : \mu = 48$ $H_1 : \mu > 48$	B1		(both)
	Since n is large, $\overline{X} \approx N\left(\mu, \frac{\sigma^2}{n}\right)$			
	(Central limit theorem)			
	$\therefore$ for H_0 true,			
	$\overline{X} \sim N\left(48, \frac{64}{100}\right) \sim N\left(48, (0.8)^2\right)$	B1		for 0.64
	$z = \frac{49.2 - 48}{0.8} = 1.50$	M1A1		
	$z_{crit} = \pm 1.6449$	B1		
	$\therefore$ do not reject H_o	A1√		on their z & critical value
	Insufficient evidence at the 5% level to. reject the committee's claim	E1√	7	on their z & critical value
(b)	$P\left(\overline{X} < 48 + 0.8 \times 1.6449\right)$	M1A1		
	$= P\left(Z < \frac{49.316 - 50}{0.8}\right)$	M1		
	$= P\left(Z < -0.8551\right)$	A1		
	$= 1 - 0.80511$ $= 0.19489$ $= 0.195$	A1	5	AWFW 0.189 to 01.98
	Total		**12**	
	Total		**60**	