General Certificate of Education
January 2005
Advanced Level Examination

MATHEMATICS (SPECIFICATION A) **MAS1/W**
Unit Statistics 1

Tuesday 25 January 2005 Morning Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 20 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAS1/W.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 Each morning, Mustafa either cycles or walks to school.

(a) The time, X minutes, taken by Mustafa to cycle to school may be modelled by a normal random variable with mean 12 and standard deviation 2.5.

Determine:

(i) $P(X < 15)$; *(2 marks)*

(ii) $P(10 < X < 15)$. *(3 marks)*

(b) On each of a random sample of 50 mornings, Mustafa records his time, y minutes, to walk to school. From his recordings, Mustafa calculates the following quantities, where $\bar{y}$ denotes the sample mean.

$$\sum y = 835.0 \qquad \sum (y - \bar{y})^2 = 533.61$$

(i) Construct a 99% confidence interval for the mean time taken by Mustafa to walk to school. *(6 marks)*

(ii) Mustafa suspects that, on average, the time he takes to walk to school is more than 25% longer than the time he takes to cycle to school.

State, with reasons, whether this suspicion is supported by your confidence interval. *(3 marks)*

2 An organisation has designed a questionnaire on health and safety which is to be distributed to a sample of 100 of its 1050 employees. Of these employees, 63 are management grades, 672 are professional grades and the remainder are administrative grades.

(a) Steve, the Safety Adviser, suggests selecting 100 employees at random from all the employees.

Write down the name for this method of selecting a random sample. *(1 mark)*

(b) Eddie, the Senior Health & Safety Adviser, suggests selecting 100 employees at random in such a way that the proportions of different grades of employee in the sample are the same as those for all employees.

(i) Write down the name for this method of selecting a random sample. *(1 mark)*

(ii) Determine the number of employees to be sampled from **each** of the three grades. *(2 marks)*

(iii) Describe how Eddie can use random numbers to select the sample of management grade employees. *(3 marks)*

(iv) To illustrate your method in part (b)(iii), use Table 13 in the formulae booklet supplied. Starting at the **first column** of the **second row**, and **working across the row**, identify the sample of management grade employees. *(2 marks)*

3 A company manufactures coloured drawing pins of which 20 per cent are yellow. The drawing pins are packed in boxes of 1000 and the contents of a box may be assumed to be a random sample.

(a) (i) Specify a probability model for Y, the number of yellow drawing pins in a box. *(1 mark)*

(ii) Use a distributional approximation to estimate $\mathrm{P}(Y \geqslant 225)$. *(6 marks)*

(b) The random variable U denotes the number of drawing pins selected from a box, without replacement, until 50 yellow drawing pins are obtained.

Give **two** reasons, in context, why U **cannot** be modelled by a binomial distribution. *(2 marks)*

4 A machine cuts lengths of water hose for washing machines. Each length cut is at least 2 metres. The extra length, X millimetres, above 2 metres may be modelled by the following probability density function, where c is a positive constant.

$$f(x) = \begin{cases} c & 0 \leqslant x \leqslant 4 \\ \frac{c}{16}(20 - x) & 4 \leqslant x \leqslant 20 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch the graph of f. *(4 marks)*

(b) By considering your sketch, or otherwise, find the exact value of c. *(3 marks)*

(c) Hence determine the probability that a randomly selected length of hose is less than 2.01 metres. *(4 marks)*

5 The probability that Janet receives at least one e-mail message on any day is 0.85. The number of messages received may be considered independent from day to day.

(a) Calculate the probability that, during a 16-day period, Janet receives at least one e-mail message on exactly 12 days. *(3 marks)*

(b) Determine the probability that, during a 30-day period, Janet receives at least one e-mail message on more than 21 days but fewer than 28 days. *(4 marks)*

6 The discrete random variable X has

$$\mathrm{E}(X) = 4 \qquad \text{and} \qquad \mathrm{E}(X^2) = 17.2.$$

(a) Find the value of $\mathrm{Var}(X)$. *(2 marks)*

(b) A circle has radius $(X + 8)$.

Find, in terms of π, values for the mean and variance of the **circumference**, C, of the circle. *(4 marks)*

(c) The **area**, S, of the circle, with radius $(X + 8)$, is to be expressed in the form

$$\pi(X^2 + aX + b),$$

where a and b are positive constants.

(i) Find values for a and b. *(2 marks)*

(ii) Hence find, in terms of π, the value of $\mathrm{E}(S)$. *(2 marks)*

END OF QUESTIONS