

General Certificate of Education
June 2005
Advanced Level Examination



MATHEMATICS (SPECIFICATION A)
Unit Pure 6

MAP6

Thursday 9 June 2005 Morning Session

In addition to this paper you will require:

- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 20 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP6.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information

- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

- 1 (a) The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

Find the matrix **AB**.

(2 marks)

- (b) The matrix **M** is given by

$$\mathbf{M} = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

- (i) Give a geometrical description of the transformation represented by the matrix **M**.
(4 marks)
- (ii) Which line is invariant under **M**?
(1 mark)

- 2 Three simultaneous equations are given by

$$x + y = a$$

$$y + 2z = b$$

$$x - y - 3z = c.$$

Find, in either order:

- (a) expressions for x , y and z in terms of a , b and c ;
(b) the inverse of the matrix

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 1 & -1 & -3 \end{bmatrix}.$$

(8 marks)

3 A transformation T is represented by the matrix \mathbf{M} given by

$$\mathbf{M} = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & -2 \\ -5 & 1 & -1 \end{bmatrix}.$$

- (a) Find the determinant of \mathbf{M} . *(2 marks)*
- (b) The volume of a given pyramid is V . Find the volume of the image of this pyramid under T . Give your answer in terms of V . *(1 mark)*

4 The matrix \mathbf{M} is given by

$$\mathbf{M} = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & 1 \\ 2 & 0 & 0 \end{bmatrix}.$$

- (a) Given that $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is an eigenvector of \mathbf{M} , find the corresponding eigenvalue λ_1 . *(2 marks)*
- (b) Given that $\lambda_2 = 4$ is an eigenvalue of \mathbf{M} , find a corresponding eigenvector. *(3 marks)*
- (c) Given that $\begin{bmatrix} 1 \\ 1 \\ a \end{bmatrix}$ is the third eigenvector of \mathbf{M} , find a and the corresponding eigenvalue λ_3 . *(5 marks)*
- (d) (i) Write down the equations of three lines which are invariant under the transformation represented by the matrix \mathbf{M} . *(2 marks)*
- (ii) Show that one of these lines is perpendicular to each of the other two. *(2 marks)*

TURN OVER FOR THE NEXT QUESTION

5 (a) Show that

$$(\mathbf{a} + \mathbf{b} - \mathbf{c}) \times (\mathbf{a} - \mathbf{b} + \mathbf{c}) = \lambda \mathbf{a} \times (\mathbf{b} - \mathbf{c}),$$

where λ is an integer to be found. (6 marks)

(b) The points O, A, B and C , where O is the origin, are all distinct. The position vectors of the points A, B and C are \mathbf{a}, \mathbf{b} and \mathbf{c} respectively. Given that

$$(\mathbf{a} + \mathbf{b} - \mathbf{c}) \times (\mathbf{a} - \mathbf{b} + \mathbf{c}) = \mathbf{0},$$

show that OA is parallel to CB . (3 marks)

6 The four points A, B, C and D have coordinates as follows:

$$A(3, -3, -5), \quad B(6, 2, -1), \quad C(7, 3, 3) \quad \text{and} \quad D(4, -2, -1).$$

(a) (i) Show that A, B, C and D are the vertices of a parallelogram. (2 marks)

(ii) Find $\overrightarrow{AB} \times \overrightarrow{AD}$. (3 marks)

(iii) Show that the area of the parallelogram is 18. (2 marks)

(b) The diagonals AC and BD of the parallelogram meet at the point K .

Explain why an equation of the line l through K perpendicular to the plane $ABCD$ is

$$\mathbf{r} = \begin{bmatrix} 5 \\ 0 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 8 \\ -4 \\ -1 \end{bmatrix}. \quad (3 \text{ marks})$$

(c) The plane Π is parallel to the plane $ABCD$ and passes through the point $L(-12, 4, 9)$. Find the coordinates of M , the point of intersection of l with Π . (6 marks)

(d) One face of a parallelepiped is $ABCD$ and the opposite face lies in the plane Π . Find the volume of the parallelepiped. (3 marks)

END OF QUESTIONS