General Certificate of Education
June 2004
Advanced Level Examination

MATHEMATICS (SPECIFICATION A) **MAP6**
Unit Pure 6

Friday 11 June 2004 Morning Session

In addition to this paper you will require: • an 8-page answer book; • the AQA booklet of formulae and statistical tables. You may use a standard scientific calculator **only**.

Time allowed: 1 hour 20 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP6.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 The lines l_1 and l_2 have equations

$$\frac{x-4}{3} = \frac{y+4}{-1} = \frac{z-4}{2}$$

and

$$\frac{x-5}{2} = \frac{y+1}{1} = \frac{z-6}{2}$$

respectively.

(a) Show that the point $(1, -3, 2)$ lies on both l_1 and l_2. *(1 mark)*

(b) Find the equation of the plane containing both l_1 and l_2, giving your answer in the form $ax + by + cz + d = 0$. *(6 marks)*

(c) Find the perpendicular distance from the origin to this plane, giving your answer in the form $k\sqrt{5}$. *(3 marks)*

2 Given that the matrix

$$\begin{bmatrix} \frac{-2\sqrt{2}}{3} & 0 & \frac{1}{3} \\ 0 & 1 & 0 \\ -\frac{1}{3} & 0 & \frac{-2\sqrt{2}}{3} \end{bmatrix}$$

represents a rotation:

(a) state the axis of rotation; *(1 mark)*

(b) find the angle of rotation, giving your answer in radians to one decimal place. *(3 marks)*

3 (a) Evaluate

$$\begin{vmatrix} 2 & a & -a \\ 1 & 3 & -2 \\ 3 & -1 & 0 \end{vmatrix},$$

giving your answer in terms of a. *(3 marks)*

(b) Determine the value of a for which the equations

$$\begin{aligned} 2x + ay - az &= 0 \\ x + 3y - 2z &= 0 \\ 3x - \ \ y \qquad &= 0 \end{aligned}$$

have solutions other than $x = y = z = 0$. *(1 mark)*

(c) In the case when $a = 1$:

(i) solve the simultaneous equations; *(3 marks)*

(ii) state the geometrical relationship between the planes represented by the equations. *(1 mark)*

4 The points A, B, C and D have coordinates $(1, 2, p)$, $(2, 4, -1)$, $(3, 1, 2)$ and $(0, -1, 4)$ respectively.

(a) Write down $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$ in terms of p. *(2 marks)*

(b) Express $(\overrightarrow{AB} \times \overrightarrow{AC}).\,\overrightarrow{AD}$ in terms of p. *(5 marks)*

(c) The parallelepiped which has AB, AC and AD as three edges has a volume of 22. Find the possible values of p. *(4 marks)*

TURN OVER FOR THE NEXT QUESTION

5 The 2×2 matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{X}$ are given by

$$\mathbf{A} = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 5 & 0 \\ 0 & -1 \end{bmatrix} \quad \text{and} \quad \mathbf{X} = \begin{bmatrix} p & q \\ r & s \end{bmatrix}.$$

(a) Express the matrices $\mathbf{AX}$ and $\mathbf{XB}$ in terms of p, q, r and s. *(3 marks)*

(b) It is given that $\mathbf{AX} = \mathbf{XB}$.

(i) Express $\mathbf{X}$ in terms of p and q only. *(4 marks)*

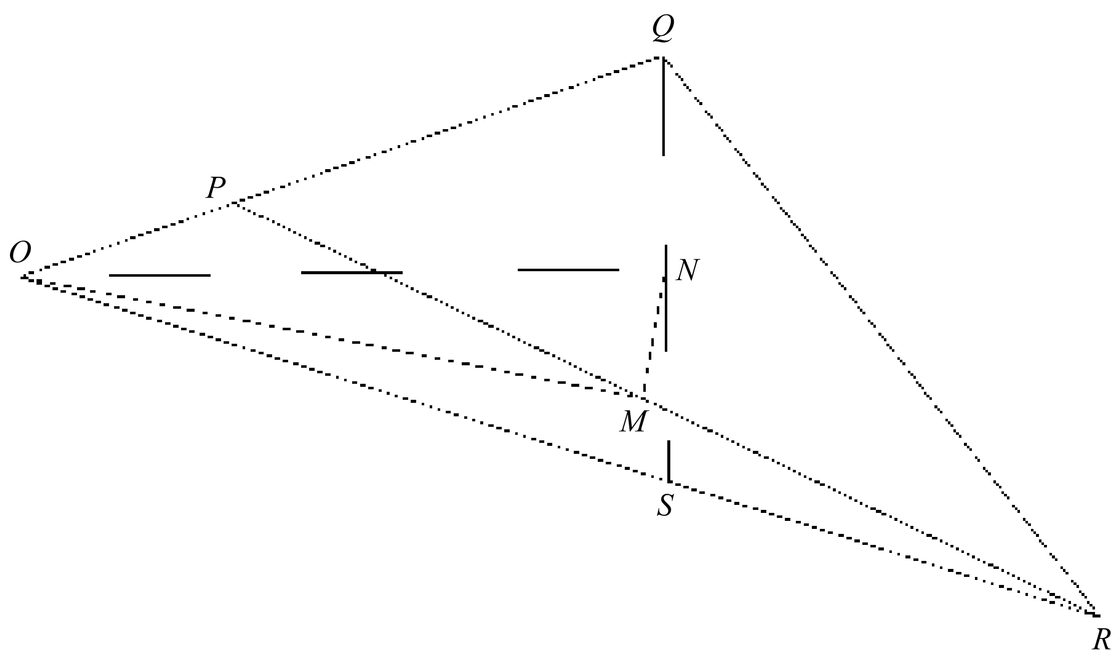
(ii) Write down the condition in terms of p and q for $\mathbf{X}^{-1}$ to exist, and express the inverse matrix $\mathbf{X}^{-1}$ in terms of p and q. *(4 marks)*

(iii) Show that, when $\mathbf{X}^{-1}$ exists, $\mathbf{X}^{-1}\mathbf{AX} = \mathbf{B}$. *(2 marks)*

(iv) Find eigenvectors and the corresponding eigenvalues of the matrix $\mathbf{A}$. *(4 marks)*

6 (a) Show that the position vector of the mid-point of the line joining the points with position vectors $\mathbf{u}$ and $\mathbf{v}$ is $\frac{1}{2}(\mathbf{u} + \mathbf{v})$. *(1 mark)*

(b) The diagram shows a triangle OQR in which $\overrightarrow{OQ} = 3\mathbf{a}$ and $\overrightarrow{OR} = 5\mathbf{b}$. The point P on OQ is such that $OP : PQ = 1 : 2$ and the point S on OR is such that $OS : SR = 3 : 2$. The points M and N are the mid-points of PR and QS respectively.



(i) Express $\overrightarrow{OM}$ and $\overrightarrow{ON}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. *(3 marks)*

(ii) Show that the area of triangle OQR is five times the area of triangle OMN. *(6 marks)*

END OF QUESTIONS