

General Certificate of Education
January 2005
Advanced Level Examination



MATHEMATICS (SPECIFICATION A)
Unit Pure 6

MAP6

Tuesday 18 January 2005 Afternoon Session

In addition to this paper you will require:

- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 20 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP6.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information

- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 The matrix \mathbf{M} is defined by

$$\mathbf{M} = \begin{bmatrix} 4 & 1 & 1 \\ 0 & 2 & -6 \\ 0 & -2 & 1 \end{bmatrix}.$$

- (a) Show that $\lambda = 4$ is an eigenvalue and find the two other eigenvalues of \mathbf{M} . (4 marks)
- (b) Verify that the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad \mathbf{v}_2 = \begin{bmatrix} -5 \\ 18 \\ 12 \end{bmatrix}$$

are eigenvectors corresponding to two of the eigenvalues. (3 marks)

- (c) Find an eigenvector \mathbf{v}_3 corresponding to the third eigenvalue. (3 marks)
- (d) Write down the image of the vector

$$\mathbf{r} = \alpha\mathbf{v}_1 + \beta\mathbf{v}_2 + \gamma\mathbf{v}_3$$

under the transformation represented by the matrix \mathbf{M} , giving your answer in the form

$$l\mathbf{v}_1 + m\mathbf{v}_2 + n\mathbf{v}_3. \quad (2 \text{ marks})$$

2 (a) Expand and simplify

$$(\mathbf{a} + 3\mathbf{b}) \times (\mathbf{a} - 2\mathbf{b}). \quad (4 \text{ marks})$$

- (b) Show that, if the vectors \mathbf{a} and \mathbf{b} are perpendicular,

$$|(\mathbf{a} + 3\mathbf{b}) \times (\mathbf{a} - 2\mathbf{b})| = k|\mathbf{a}||\mathbf{b}|,$$

where k is an integer.

(2 marks)

3 The matrix \mathbf{A} is given by

$$\mathbf{A} = \begin{bmatrix} a & 5 & 4 \\ 4 & a & 0 \\ 3 & a & 1 \end{bmatrix},$$

where a is a constant.

(a) Express $\det \mathbf{A}$ in terms of a . (3 marks)

(b) The non-singular matrix \mathbf{B} is such that

$$\det (\mathbf{AB}) = \det \mathbf{B}.$$

Find the possible values of a . (3 marks)

4 The vectors \mathbf{u}_1 , \mathbf{u}_2 and \mathbf{u}_3 are given by

$$\mathbf{u}_1 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} \quad \text{and} \quad \mathbf{u}_3 = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}.$$

(a) (i) Find $(\mathbf{u}_1 \times \mathbf{u}_2) \cdot \mathbf{u}_3$. (3 marks)

(ii) Explain why it is **not** possible to express \mathbf{u}_3 as a linear combination of \mathbf{u}_1 and \mathbf{u}_2 . (2 marks)

(b) The vector \mathbf{u}_4 is given by

$$\mathbf{u}_4 = \begin{bmatrix} 2 \\ 7 \\ 2 \end{bmatrix}.$$

(i) Find a vector \mathbf{u} such that

$$\mathbf{u}_4 = \begin{bmatrix} 2 & 3 & 1 \\ 1 & -2 & 3 \\ 1 & 1 & 1 \end{bmatrix} \mathbf{u}. \quad (6 \text{ marks})$$

(ii) Hence, or otherwise, express \mathbf{u}_4 as a linear combination of \mathbf{u}_1 , \mathbf{u}_2 and \mathbf{u}_3 . (2 marks)

5 The points A , B and C have coordinates $(3, 4, 1)$, $(-2, 1, 4)$ and $(1, 2, -1)$ respectively.

(a) Find:

(i) $\overrightarrow{AB} \times \overrightarrow{AC}$; (3 marks)

(ii) the exact value of the area of triangle ABC ; (2 marks)

(iii) the Cartesian equation of the plane Π containing A , B and C . (2 marks)

(b) The line l passes through the point $D(0, -5, 0)$ and is perpendicular to Π . Find:

(i) the equation of l in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$; (1 mark)

(ii) the coordinates of the point of intersection of l with Π . (3 marks)

(c) Deduce the volume of the tetrahedron $ABCD$. (3 marks)

[The volume of a tetrahedron is $\frac{1}{3}$ area of base \times height.]

6 (a) The transformation T_1 represented by the matrix

$$\mathbf{M}_1 = \begin{bmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

is an anticlockwise rotation about the origin. Find the angle of rotation. (2 marks)

(b) The transformation T_2 represented by the matrix

$$\mathbf{M}_2 = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$$

is a reflection in the line $y = mx$. Find the value of m . (3 marks)

(c) (i) The matrix \mathbf{M}_3 is given by $\mathbf{M}_3 = \mathbf{M}_1\mathbf{M}_2$. Find \mathbf{M}_3 . (2 marks)

(ii) The matrix \mathbf{M}_3 represents a single transformation T_3 . Give a geometrical description of T_3 . (2 marks)

END OF QUESTIONS