

General Certificate of Education
January 2004
Advanced Level Examination



MATHEMATICS (SPECIFICATION A)
Unit Pure 6

MAP6

Friday 16 January 2004 Afternoon Session

In addition to this paper you will require:

- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 20 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP6.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information

- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 The position vectors of three points A , B and C relative to an origin O are given by

$$\mathbf{a} = 3\mathbf{i} + 4\mathbf{j} - 2\mathbf{k},$$

$$\mathbf{b} = 2\mathbf{i} + 3\mathbf{j},$$

and

$$\mathbf{c} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$$

respectively.

(a) Find:

(i) $\mathbf{a} \times \mathbf{b}$;

(2 marks)

(ii) $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$.

(2 marks)

(b) State a geometrical relationship between the points O , A , B and C .

(1 mark)

2 (a) Find the value of the determinant

$$\begin{vmatrix} 2 & 3 & -2 \\ 1 & -1 & 0 \\ 0 & -1 & 2 \end{vmatrix}.$$

(2 marks)

(b) Determine whether the vectors

$$\mathbf{u} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 3 \\ -1 \\ -1 \end{bmatrix} \quad \text{and} \quad \mathbf{w} = \begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix}$$

are linearly dependent.

(1 mark)

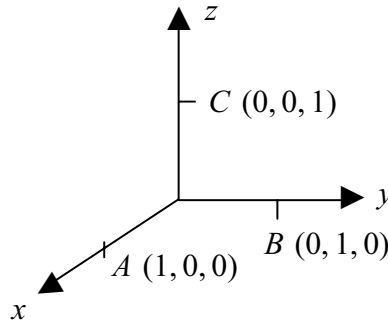
(c) Express the vector $\begin{bmatrix} 0 \\ 3 \\ -2 \end{bmatrix}$ as a linear combination of \mathbf{u} , \mathbf{v} and \mathbf{w} .

(6 marks)

3 A matrix \mathbf{M}_1 is given by

$$\mathbf{M}_1 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

- (a) Give a geometrical description of the transformation T_1 represented by \mathbf{M}_1 . (2 marks)
- (b) The diagram below shows the points $A(1, 0, 0)$, $B(0, 1, 0)$ and $C(0, 0, 1)$.



A second transformation, T_2 , is a rotation of π radians about the line $x = z, y = 0$.

- (i) Find the images of the points A , B , and C under T_2 . (2 marks)
- (ii) Write down the matrix \mathbf{M}_2 which represents this transformation. (2 marks)
- (c) (i) Show that the matrix \mathbf{M}_3 , which represents the transformation T_2 followed by the transformation T_1 , is given by

$$\mathbf{M}_3 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (2 \text{ marks})$$

- (ii) Give a geometrical description of the transformation represented by the matrix \mathbf{M}_3 . (2 marks)

TURN OVER FOR THE NEXT QUESTION

4 The planes Π_1 and Π_2 have equations

$$x + 2y - z = 1$$

and

$$x + 3y + z = 6$$

respectively.

- (a) Verify that the point P , with coordinates $(1, 1, 2)$, lies on both planes. *(1 mark)*
- (b) Find the equation of l , the line of intersection of Π_1 and Π_2 , giving your answer in the form $\frac{x - \alpha}{p} = \frac{y - \beta}{q} = \frac{z - \gamma}{r}$. *(5 marks)*
- (c) Find the acute angle between the line l and the line through P which is parallel to the y -axis. Give your answer to the nearest 0.1 of a degree. *(3 marks)*

5 The matrices \mathbf{A} and \mathbf{B} are given by

$$\mathbf{A} = \begin{bmatrix} 3 & -1 & p \\ 0 & -5 & p \end{bmatrix}$$

and

$$\mathbf{B} = \begin{bmatrix} p & -1 \\ -2 & 0 \\ -3 & 3 \end{bmatrix}.$$

- (a) Find \mathbf{AB} in terms of p . *(3 marks)*
- (b) Given that $\det \mathbf{AB} = 0$:
- (i) find the possible values of p ; *(4 marks)*
- (ii) for each value of p , find the matrix $\mathbf{B}^T \mathbf{A}^T$; *(3 marks)*
- (iii) explain why $(\mathbf{AB})^{-1}$ does not exist. *(1 mark)*

6 The matrix \mathbf{M} is defined by

$$\mathbf{M} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 1 & 3 \end{bmatrix}.$$

- (a) Verify that \mathbf{M} has an eigenvalue 1 and find a corresponding eigenvector. (5 marks)
- (b) (i) Show that \mathbf{M} has only one other distinct eigenvalue and find this eigenvalue. (4 marks)
- (ii) Deduce that any non-zero vector of the form $\begin{bmatrix} p \\ q \\ q \end{bmatrix}$, where p and q are real, is an eigenvector corresponding to this eigenvalue. (3 marks)
- (c) (i) Find a Cartesian equation of a line l_1 lying in the plane $x = 0$ such that each point of l_1 is invariant under the transformation T represented by \mathbf{M} . (2 marks)
- (ii) Find a Cartesian equation of another line l_2 in the plane $x = 0$ which is invariant under T . (2 marks)

END OF QUESTIONS