Version 1.0: 0805

General Certificate of Education

Mathematics 6300
Specification A

MAP6 Pure 6

Mark Scheme

2005 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key to Mark Scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m marks and is for	accuracy
B	mark is independent of M or m marks and is for	accuracy
E	mark is for	explanation
√or ft or F		follow through from previous incorrect result
CAO		correct answer only
AWFW		anything which falls within
AWRT		anything which rounds to
AG		answer given
SC		special case
OE		or equivalent
A2,1		2 or 1 (or 0) accuracy marks
–*x* EE		deduct *x* marks for each error
NMS		no method shown
PI		possibly implied
SCA		substantially correct approach
c		candidate
sf		significant figure(s)
dp		decimal place(s)

Abbreviations used in Marking

MC – *x*	deducted *x* marks for mis-copy
MR – *x*	deducted *x* marks for mis-read
ISW	ignored subsequent working
BOD	given benefit of doubt
WR	work replaced by candidate
FB	formulae book

Application of Mark Scheme

No method shown:	
Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise
More than one method / choice of solution:	
2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only
Crossed out work	do not mark unless it has not been replaced
Alternative solution using a correct or partially correct method	award method and accuracy marks as appropriate

MAP6

Q	Solution	Marks	Total	Comments
1(a)	$\mathbf{AB} = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$	B2,1,0	2	
(b)(i)	This transformation represents a rotation of 180°	B1		accept reflection in z-axis or in both x- and y-axes
	about the z-axis	B1		
	together with an enlargement scale factor 3	B1		accept 'stretch' as long as clear, but not enlargement along an axis
	from the origin	B1	4	
(ii)	z-axis	B1	1	OE
	Total		**7**	
2	**Method 1:**			
	$x - 2z = a - b$	M1A1		2 equations in the same 2 unknowns
	$x - z = b + c$	A1		**Method 2:**
	Solving: $z = -a + 2b + c$	A1F		determinant of matrix B1
	$y = 2a - 3b - 2c$	A1F		cofactors M1
	$x = -a + 3b + 2c$	A1F		transpose M1
	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 & 3 & 2 \\ 2 & -3 & -2 \\ -1 & 2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$	M1		inverse matrix $\begin{bmatrix} -1 & 3 & 2 \\ 2 & -3 & -2 \\ -1 & 2 & 1 \end{bmatrix}$ A2,1,0 ft on determinant **SC** consistent sign error in cofactors: mark B1M1A1
	Inverse is $\begin{bmatrix} -1 & 3 & 2 \\ 2 & -3 & -2 \\ -1 & 2 & 1 \end{bmatrix}$	A1F	8	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 & 3 & 2 \\ 2 & -3 & -2 \\ -1 & 2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ M1A1F $x = -a + 3b + 2c$ $y = 2a - 3b - 2c$ A1F $z = -a + 2b + c$ **Note:** if both methods used, mark the better of the two and use the rest of the available marks for the other part
	Total		**8**	
3(a)	det $\mathbf{M}$ = +1(–1 + 2) + 2(–2 –10) + 3(2 + 5)	M1		allow one error
	= – 2	A1	2	
(b)	New volume = $2V$	A1F	1	ft here provided new volume > 0 and determinant < 0
	Total		**3**	

MAP6 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$\begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & 1 \\ 2 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\lambda_1 = 2$	M1A1	2	
(b)	$\begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & 1 \\ 2 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 4\begin{bmatrix} x \\ y \\ z \end{bmatrix}$	M1		
	$+2x + y - z = 0$ $2x \quad -4z = 0$	A1		
	$\begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$	A1F	3	
(c)	$\begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & 1 \\ 2 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ a \end{bmatrix} = \lambda_3\begin{bmatrix} 1 \\ 1 \\ a \end{bmatrix}$	M1		if $\begin{vmatrix} 2-\lambda & -1 & 1 \\ -2 & 3-\lambda & 1 \\ 2 & 0 & -\lambda \end{vmatrix}$ used: $-\lambda^3 + 5\lambda^2 - 2\lambda - 8 = 0$ M1 $\lambda_3 = -1$ A1 $a = -2$ M1A1
	$1 + a = \lambda_3$ $2 = \lambda_3 a$	A1		
	elimination of one letter: $1 + a = \dfrac{2}{a}$	M1		
	$(a+2)(a-1) = 0$	A1F		provided quadratic factorises
	$a = -2 \quad (a \neq 1) \quad \lambda_3 = -1$	A1F	5	
(d)(i)	$\mathbf{r} = \lambda\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} \quad \mathbf{r} = \mu\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \mathbf{r} = v\begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$	B2,1,0F	2	no **r** = B1 max
(ii)	$\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}\cdot\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0$	B1		
	$\begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}\cdot\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0$	B1	2	
	Total		**14**	

MAP6 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$\mathbf{a}\times\mathbf{a}-\mathbf{a}\times\mathbf{b}+\mathbf{a}\times\mathbf{c}$ $+\mathbf{b}\times\mathbf{a}-\mathbf{b}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}$ $-\mathbf{c}\times\mathbf{a}+\mathbf{c}\times\mathbf{b}-\mathbf{c}\times\mathbf{c}$	M1A1		$\mathbf{a}^2-\mathbf{ab}$ etc M0A0
	Use of $\mathbf{a}\times\mathbf{a}=0$	M1		PI ie allow $\mathbf{a}^2=0$ } for the M marks
	$\mathbf{a}\times\mathbf{b}=-\mathbf{b}\times\mathbf{a}$	M1		PI $\mathbf{ab}=-\mathbf{ba}$ } for the M marks but M1M1 **only** in this case
	$-2\mathbf{a}\times\mathbf{b}+2\mathbf{a}\times\mathbf{c}$	A1		OE
	$-2\mathbf{a}\times(\mathbf{b}-\mathbf{c})$	A1	6	
(b)	$\overrightarrow{OA}=\mathbf{a}$ and $\overrightarrow{CB}=\mathbf{b}-\mathbf{c}$	B1		
	$\mathbf{a}\neq 0$, $\mathbf{b}-\mathbf{c}\neq 0$ (given)	E1		
	$\mathbf{a}\times(\mathbf{b}-\mathbf{c})=0\Rightarrow OA$ parallel to CB	B1	3	
	Total		**9**	

MAP6 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$\overrightarrow{AB} = \overrightarrow{DC} = \begin{bmatrix} 3 \\ 5 \\ 4 \end{bmatrix}$ $\therefore$ $AB \parallel DC$ and $AB = DC$ $\therefore$ parallelogram	B1B1	2	OE eg $AB \parallel CD$ and $AD \parallel BC$ or $AB = DC$ and $AD = BC$
(ii)	$\overrightarrow{AB} \times \overrightarrow{AD} = \begin{bmatrix} 3 \\ 5 \\ 4 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}$	B1		
	$= \begin{bmatrix} 16 \\ -8 \\ -2 \end{bmatrix}$	M1A1F	3	
(iii)	$\left\lvert \overrightarrow{AB} \times \overrightarrow{AD} \right\rvert = \sqrt{16^2 + 8^2 + 2^2} = 18$	M1A1	2	AG
(b)	K is (5, 0, – 1)	M1A1		**must** show a method here
	direction of l is $\begin{bmatrix} 8 \\ -4 \\ -1 \end{bmatrix}$	B1	3	
(c)	Π has equation $8x - 4y - z = -121$	M1A1		
	At M, $8(5 + 8\lambda) - 4(-4\lambda) - (-1 - \lambda) = -121$	M1A1F		
	$\lambda = -2$	A1F		
	M is (–11, 8, 1)	A1	6	CAO
(d)	KM is distance between $(5, 0, -1)$ and $(-11, 8, 1)$			
	$\sqrt{16^2 + 8^2 + (-2)^2} = 18$	M1A1		clear method
	$\therefore$ Volume $= 18 \times 18 = 324$	A1F	3	
	Total		**19**	
	TOTAL		**60**	