GCE 2004
June Series

ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark Scheme

Mathematics A
Unit MAP6

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Key to Mark Scheme

M...mark is for ... method
m ...mark is dependent on one or more M marks and is for... method
A...mark is dependent on M or m marks and is for ... accuracy
B...mark is independent of M or m marks and is for ...method and accuracy
E...mark is for...explanation
√ or ft or F... follow through from previous incorrect result
CAO...correct answer only
AWFW ...anything which falls within
AWRT ...anything which rounds to
AG ... answer given
SC ... special case
OE... or equivalent
A2,1... 2 or 1 (or 0) accuracy marks
-*x* EE...deduct *x* marks for each error
NMS...no method shown
PI ...possibly implied
SCA ...substantially correct approach
c... candidate
SF...significant figure(s)
DP ... decimal place(s)

Abbreviations used in Marking

MC – *x*... deducted *x* marks for mis-copy
MR – *x*...deducted *x* marks for mis-read
ISW... ignored subsequent working
BOD...given benefit of doubt
WR... work replaced by candidate
FB ... formulae booklet

Application of Mark Scheme

No method shown:
Correct answer without working...mark as in scheme
Incorrect answer without working ... zero marks unless specified otherwise

More than one method/choice of solution:

2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only
Crossed out work	do not mark unless it has not been replaced
Alternative solution using a correct or partially correct method	award method and accuracy marks as appropriate

MAP6

Q	Solution	Marks	Total	Comments
1(a)	$\frac{1-4}{3}=\frac{-3+4}{-1}=\frac{2-4}{2}=-1$	B1	1	all three must be seen
	$\frac{1-5}{2}=\frac{-3+1}{1}=\frac{2-6}{2}=-2$			
(b)	$\begin{bmatrix}3\\-1\\2\end{bmatrix}\times\begin{bmatrix}2\\1\\2\end{bmatrix}$	M1A1		(b) **Alternative**:- $\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\-3\\2\end{bmatrix}+\lambda\begin{bmatrix}3\\-1\\2\end{bmatrix}+\mu\begin{bmatrix}2\\1\\2\end{bmatrix}$ M1
	$=\begin{bmatrix}-4\\-2\\5\end{bmatrix}$	A1F		ft miscopy $x=1+3\lambda+2\mu$, $y=-3-\lambda+\mu$ A1, $z=2+2\lambda+2\mu$
	$\begin{bmatrix}x\\y\\z\end{bmatrix}\bullet\begin{bmatrix}-4\\-2\\5\end{bmatrix}=\begin{bmatrix}1\\-3\\2\end{bmatrix}\bullet\begin{bmatrix}-4\\-2\\5\end{bmatrix}$	M1A1F		eliminate λ M1A1F eliminate μ A1F result A1F
	Equation of plane is $4x+2y-5z+12=0$	A1F	6	
(c)	Perpendicular distance from $(0,0,0)$			(c) **Alternative**
	$=\frac{12}{\sqrt{4^2+2^2+(-5)^2}}$	M1A1F		$\overrightarrow{OP}=-\frac{4}{15}\begin{bmatrix}4\\2\\-5\end{bmatrix}$ M1A1F
	$=\frac{4}{5}\sqrt{5}$	A1	3	cao $=\frac{4\sqrt{5}}{5}$ A1 cao
	Total		**10**	

MAP6 (Cont)

Q	Solution	Marks	Total	Comments
2(a)	y-axis	B1	1	
(b)	$\sin\theta = \frac{1}{3}$, $\cos\theta = \frac{-2\sqrt{2}}{3}$	B1B1		Correct answer with $\tan\theta = -\frac{1}{2\sqrt{2}}$ scores 3 marks
	angle is $\pi - \sin^{-1}\frac{1}{3} = 2.8$	B1	3	B0 here if B0 awarded in line above cao from correct $\cos\theta$ and $\sin\theta$ 2.8 with no method B1 3.5 as an answer could be correct but needs scrutiny
	Total		**4**	
3(a)	$\Delta = 2(0-2) - a(0+6) - a(-1-9)$	M1A1		M1 for correct method of expansion
	$= 4a - 4$	A1F	3	ft on one error
(b)	$a = 1$	B1F	1	
(c)(i)	$x = t$, $y = 3t$	M1A1		M1 for complete method
	$z = 5t$	A1F	3	If answer given as $x = \frac{1}{3}y = \frac{1}{5}z$ o.e. deduct 1 mark **Alternative** $\begin{bmatrix}1\\3\\5\end{bmatrix}$ B1 $\lambda\begin{bmatrix}1\\3\\5\end{bmatrix}$ M1A1F
(ii)	sheaf (oe) of planes	E1	1	
	Total		**8**	

MAP6 (Cont)

Q	Solution	Marks	Total	Comments
4(a)	$\overrightarrow{AB}=\begin{bmatrix}1\\2\\-1-p\end{bmatrix}$ $\overrightarrow{AC}=\begin{bmatrix}2\\-1\\2-p\end{bmatrix}$			
	$\overrightarrow{AD}=\begin{bmatrix}-1\\-3\\4-p\end{bmatrix}$	B2, 1, 0	2	
(b)	$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{bmatrix}2(2-p)\ +(-1-p)\\-(2-p)+2(-1-p)\\-5\end{bmatrix}$	M1A1F		Alternative $\begin{vmatrix}-1 & -3 & 4-p\\1 & 2 & -1-p\\2 & -1 & 2-p\end{vmatrix}$
	$=\begin{bmatrix}3-3p\\-4-p\\-5\end{bmatrix}$	A1F		expanded M1 correctly A2, 1, 0 gather terms m1 $11p-11$ A1F
	$(\overrightarrow{AB}\times\overrightarrow{AC}).\overrightarrow{AD}=\begin{bmatrix}3-3p\\-4-p\\-5\end{bmatrix}\begin{bmatrix}-1\\-3\\4-p\end{bmatrix}$			
	$=-11+11p$	M1A1F	5	
(c)	$\lvert-11+11p\rvert=22$			
	$p=3$	M1A1F		Incorrect formula M0 here
	$p=-1$	M1A1F	4	but allow this M1 even if formula is incorrect, and A1F also
	Total		**11**	

MAP6 (Cont)

Q	Solution	Marks	Total	Comments
5(a)	$\mathbf{AX}=\begin{bmatrix}3&2\\4&1\end{bmatrix}\begin{bmatrix}p&q\\r&s\end{bmatrix}=\begin{bmatrix}3p+2r&3q+2s\\4p+r&4q+s\end{bmatrix}$	M1A1		M1 for method of multiplying matrices
	$\mathbf{XB}=\begin{bmatrix}p&q\\r&s\end{bmatrix}\begin{bmatrix}5&0\\0&-1\end{bmatrix}=\begin{bmatrix}5p-q\\5r-s\end{bmatrix}$	B1	3	
(b)(i)	$\mathbf{AX}=\mathbf{XB}$ $3p+2r=5p,\ 4p+r=5r$ $3q+2s=-q,\ 4q+s=-s$	M1A1F		2 equations are sufficient
	$p=r,\ -2q=s$	A1		cao
	$\mathbf{X}=\begin{bmatrix}p&q\\p&-2q\end{bmatrix}$	A1F	4	
(ii)	Det $\mathbf{X}=-3pq\neq 0$	B1F		Any valid unsimplified expression $\neq 0$
	$\mathbf{X}^{-1}=-\dfrac{1}{3pq}\begin{bmatrix}-2q&-q\\-p&p\end{bmatrix}$	M1		For method of finding inverse
		m1		Appropriate use of determinant
		A1F	4	
(iii)	$\mathbf{X}^{-1}\mathbf{AX}=\mathbf{X}^{-1}\mathbf{XB}=\mathbf{IB}=\mathbf{B}$	M1A1	2	or directly (i.e. from original matrices) $\mathbf{X}^{-1}\mathbf{X}=\mathbf{I}$ must be seen
(iv)	Eigenvectors $\begin{bmatrix}1\\1\end{bmatrix}$ $\begin{bmatrix}1\\-2\end{bmatrix}$	B1B1		OE deduct B1 once if eigenvectors and eigenvalues are not clearly corresponding
	Eigenvalues $5,-1$	B1B1	4	
	Total		**17**	

MAP6 (Cont)

Q	Solution	Marks	Total	Comments
6(a)	Any method	B1	1	Must be convincing
(b)(i)	$\overrightarrow{OM} = \frac{1}{2}(\mathbf{a} + 5\mathbf{b})$	M1A1		M1 method for either
	$\overrightarrow{ON} = \frac{1}{2}(3\mathbf{a} + 3\mathbf{b})$	A1	3	
(ii)	$\Delta OMN = \frac{1}{2}\left\lvert\overrightarrow{OM} \times \overrightarrow{ON}\right\rvert$			
	$= \frac{1}{8}\lvert(\mathbf{a} + 5\mathbf{b}) \times (3\mathbf{a} + 3\mathbf{b})\rvert$	M1		M0 if modules sign missing
	Use of $\mathbf{a} \times \mathbf{a} = 0$	B1		
	Use of $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$	B1		
	$\Delta OMN = 1.5\lvert\mathbf{a} \times \mathbf{b}\rvert$	A1F		Must score both B1 s for this A1
	$\Delta OQR = \frac{1}{2}\lvert 3\mathbf{a} \times 5\mathbf{b}\rvert$	B1		
	$\Delta OQR = 5\ \Delta OMN$	A1	6	CAO
	Total		**10**	
	Total		**60**	