

GCE 2005

January Series



Mark Scheme

Mathematics A

(MAP6)

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

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Key to Mark Scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m marks and is for	accuracy
B	mark is independent of M or m marks and is for	method and accuracy
E	mark is for	explanation
√ or ft or F	follow through from previous	incorrect result
CAO	correct answer only	
AWFW	anything which falls within	
AWRT	anything which rounds to	
AG	answer given	
SC	special case	
OE	or equivalent	
A2,1	2 or 1 (or 0) accuracy marks	
-x EE	deduct x marks for each error	
NMS	no method shown	
PI	possibly implied	
SCA	substantially correct approach	
c	candidate	
SF	significant figure(s)	
DP	decimal place(s)	

Abbreviations used in Marking

MC – x	deducted x marks for mis-copy
MR – x	deducted x marks for mis-read
ISW	ignored subsequent working
BOD	given benefit of doubt
WR	work replaced by candidate
FB	formulae booklet

Application of Mark Scheme

No method shown:

Correct answer without working	mark as in scheme
Incorrect answer without working.....	zero marks unless specified otherwise

More than one method/choice of solution:

2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only

Crossed out work

do not mark unless it has not been replaced

Alternative solution using a correct or partially correct method

award method and accuracy marks as appropriate

MAP6

Q	Solution	Marks	Total	Comments
1(a)	$\begin{bmatrix} 4-\lambda & 1 & 1 \\ 0 & 2-\lambda & -6 \\ 0 & -2 & 1-\lambda \end{bmatrix}$ $=(4-\lambda)[(2-\lambda)(1-\lambda)-12]=0$ $\lambda=4$ $\lambda=5,-2$	M1A1 B1 A1F	4	Allow at any stage
(b)	$\begin{bmatrix} 4 & 1 & 1 \\ 0 & 2 & -6 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix} = 4 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 4 & 1 & 1 \\ 0 & 2 & -6 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} -5 \\ 18 \\ 12 \end{bmatrix}$ $= \begin{bmatrix} 10 \\ -36 \\ 24 \end{bmatrix} = -2 \begin{bmatrix} -5 \\ 18 \\ 12 \end{bmatrix}$	M1A1 A1	3	
(c)	$\begin{bmatrix} -1 & 1 & 1 \\ 0 & -3 & -6 \\ 0 & -2 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -x-y-z \\ -3y-6z \\ -2y-4z \end{bmatrix}$ <p>eigenvector $\mathbf{v}_3 = \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$</p>	M1A1F A1	3	OE
(d)	$\mathbf{r}^1 = 4\alpha\mathbf{v}_1 - 2\beta\mathbf{v}_2 + 5\gamma\mathbf{v}_3$	M1A1F	2	
Total			12	

MAP6 (cont)

Q	Solution	Marks	Total	Comments
2(a)	$\mathbf{a} \times \mathbf{a} + 3\mathbf{b} \times \mathbf{a} - \mathbf{a} \times 2\mathbf{b} - 3\mathbf{b} \times 2\mathbf{b}$ Use of $\mathbf{a} \times \mathbf{a} = 0$ Use of $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$ Result $5\mathbf{b} \times \mathbf{a}$	M1 B1 B1 A1	4	or $\mathbf{b} \times \mathbf{b} = 0$ PI PI or $-5\mathbf{a} \times \mathbf{b}$
(b)	\mathbf{a} perpendicular $\mathbf{b} \Rightarrow \mathbf{a} \times \mathbf{b} = \mathbf{a} \mathbf{b} $ Result $5 \mathbf{a} \mathbf{b} $	M1 A1F	2	
Total			6	
3(a)	$\det \mathbf{A} = a(a-0) - 5(4-0) + 4(4a-3a)$ $= a^2 + 4a - 20$	M1A1 A1F	3	
(b)	$\det \mathbf{AB} = \det \mathbf{B} \Rightarrow \det \mathbf{A} = 1$ $a^2 + 4a - 20 = 1$ $a = -7, 3$	B1 M1 A1F	3	PI $a^2 + 4a - 20 = 0$ M0
Total			6	

MAP6 (cont)

Q	Solution	Marks	Total	Comments	
4(a)(i)	$\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \times \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ -7 \end{bmatrix}$	M1A1		If determinant method used allow M1A2, 1, 0	
	$\begin{bmatrix} 3 \\ 1 \\ -7 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} = -1$	A1F	3		
(ii)	<p>Sensible reasons e.g.</p> $(\mathbf{u}_1 \times \mathbf{u}_2) \cdot \mathbf{u}_3 \neq 0$ $\therefore \mathbf{u}_3 \neq \alpha \mathbf{u}_1 + \beta \mathbf{u}_2 / \text{not coplanar}$	B2,1,0	2		
(b)(i)	$\begin{bmatrix} 2 \\ 7 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 1 \\ 1 & -2 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$			Alternative using inverse matrix	
	One equation with 2 unknowns			Determinant M B1	
	e.g. $a + 2b = 0$	M1A1		Inverse matrix	
	Second equation eg $5a + 11b = -1$	A1		$\mathbf{M}^{-1} = \frac{-1}{1} \begin{bmatrix} -5 & -2 & 11 \\ 2 & 1 & -5 \\ 3 & 1 & -7 \end{bmatrix}$	M1A2,1,0
	Solve simultaneously $a = 2, b = -1$ $c = 1$	M1 A1F A1F	6	$\mathbf{M}^{-1} \begin{bmatrix} 2 \\ 7 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$	M1A1F
(ii)	$\begin{bmatrix} 2 \\ 7 \\ 2 \end{bmatrix} = 2 \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} - 1 \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} + 1 \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$	M1A1F	2		
Total			13		

MAP6 (cont)

Q	Solution	Marks	Total	Comments
5(a)(i)	$\overrightarrow{AB} = \begin{bmatrix} -5 \\ -3 \\ 3 \end{bmatrix}$ $\overrightarrow{AC} = \begin{bmatrix} -2 \\ -2 \\ -2 \end{bmatrix}$	B1		
	$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{bmatrix} 12 \\ -16 \\ 4 \end{bmatrix}$	M1A1	3	ft incorrect \overrightarrow{AB} or \overrightarrow{AC}
(ii)	$\Delta ABC = \frac{1}{2} \overrightarrow{AB} \times \overrightarrow{AC} = 2\sqrt{26}$	M1A1F	2	OE
(iii)	Π is $12x - 16y + 4z = -24$ $(3x - 4y + z = -6)$	M1A1	2	OE
(b)(i)	Equation of l is $\mathbf{r} = \begin{bmatrix} 0 \\ -5 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} 3 \\ -4 \\ 1 \end{bmatrix}$	B1	1	OE ft incorrect $\begin{bmatrix} 3 \\ -4 \\ 1 \end{bmatrix}$
(ii)	meets Π where $3(3\lambda) - 4(-4\lambda - 5) + \lambda = -6$ $\lambda = -1$ point is $(-3, -1, -1)$	M1 A1F A1F	3	
(c)	height is $\sqrt{(3^2 + 4^2 + 1^2)} = \sqrt{26}$	B1		
	Volume of $ABCD$ is $\frac{2\sqrt{26} \sqrt{26}}{3}$ $= \frac{52}{3}$	M1 A1	3	Vol = $\frac{1}{3} \mathbf{a} \times \mathbf{b} \cdot \mathbf{c} $ allow B1 CAO
Total			14	

MAP6 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$\cos \theta = -\frac{\sqrt{3}}{2}, \sin \theta = \frac{1}{2}$	M1	2	Correct answer without adequate working B1 or $\frac{5\pi}{6}$
	$\theta = 150^\circ$	A1		
(b)	$\cos 2\phi = \frac{1}{2}, \sin 2\phi = -\frac{\sqrt{3}}{2}$	M1	3	or $2\phi = -\frac{\pi}{3}$ ft simple error: answer must not contain trig functions
	$2\phi = -60$	A1		
	$y = -\tan 30x = -\frac{1}{\sqrt{3}}x$	A1		
(c)(i)	$M_3 =$ $\begin{bmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$	M1A1	2	
	$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$			
(ii)	reflection	B1	2	
	in $y = x$	B1		
	Total		9	
	Total		60	