ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark scheme January 2004

GCE

Mathematics A

Unit MAP6

Key to mark scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m mark and is for	accuracy
B	mark is independent of M or m marks and is for	method and accuracy
E	mark is for	explanation
√ or ft or F		follow through from previous incorrect result
CAO		correct answer only
AWFW		anything which falls within
AWRT		anything which rounds to
AG		answer given
SC		special case
OE		or equivalent
A2,1		2 or 1 (or 0) accuracy marks
– *x* EE		Deduct *x* marks for each error
NMS		No method shown
PI		Perhaps implied
c		Candidate

Abbreviations used in marking

MC – *x*	deducted *x* marks for miscopy
MR – *x*	deducted *x* marks for misread
ISW	ignored subsequent working
BOD	gave benefit of doubt
WR	work replaced by candidate

Application of mark scheme

Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise

Award method and accuracy marks as appropriate to an alternative solution using a correct method or partially correct method.

Q	Solution	Marks	Total	Comments
1 (a)(i)	$\mathbf{a}\times\mathbf{b}=\begin{bmatrix}3\\4\\-2\end{bmatrix}\times\begin{bmatrix}2\\3\\0\end{bmatrix}=\begin{bmatrix}6\\-4\\1\end{bmatrix}$	M1A1	2	
(ii)	$(\mathbf{a}\times\mathbf{b})\bullet\mathbf{c}=\begin{bmatrix}6\\-4\\1\end{bmatrix}\bullet\begin{bmatrix}1\\2\\2\end{bmatrix}=6-8+2=0$	M1A1F	2	
(b)	*O, A, B* and *C* and are coplanar	E1	1	no ft here
	Total		**5**	
2 (a)	$\triangle = 2\times(-2)-3(2)-2\times(-1)=-8$	M1A1	2	
(b)	Independent since $\triangle \neq 0$	E1	1	
(c)	$0=2\alpha+3\beta-2\gamma$ $3=\alpha-\beta$ $-2=-\beta+2\gamma$	M1A1		
	Two simultaneous equations in two unknowns	M1		
	Solution for two unknowns	A1FA1F		
	Third unknown	A1F	6	
	$(\alpha=1,\ \beta=-2,\ \gamma=-2)$			
	Total		**9**	

Q	Solution	Marks	Total	Comments
3 (a)	$\mathbf{M}_1$ is a rotation of $-\dfrac{\pi}{2}$ about y-axis	B1B1	2	Accept $-\dfrac{\pi}{2}$, 90°
(b)(i)	$(1,\ 0,\ 0)\rightarrow(0,\ 0,\ 1)$ $(0,\ 1,\ 0)\rightarrow(0,-1,\ 0)$ $(0,\ 0,\ 1)\rightarrow(1,\ 0,\ 0)$	B2,1,0	2	
(ii)	Matrix $\mathbf{M}_2=\begin{bmatrix}0&0&1\\0&-1&0\\1&0&0\end{bmatrix}$	M1A1F	2	
(c)(i)	$\begin{bmatrix}0&0&-1\\0&1&0\\1&0&0\end{bmatrix}\begin{bmatrix}0&0&1\\0&-1&0\\1&0&0\end{bmatrix}=\begin{bmatrix}-1&0&0\\0&-1&0\\0&0&1\end{bmatrix}$	M1A1	2	AG M1 for getting the order of the matrices correct
(ii)	Rotation of π about the z-axis	B1B1	2	Accept 180°
	Total		**10**	
4 (a)	$1+2-2=1,\quad 1+3+2=6$	B1	1	
(b)	$\begin{bmatrix}1\\2\\-1\end{bmatrix}\times\begin{bmatrix}1\\3\\1\end{bmatrix}$	M1A1		Alternative method for 4(b) Elimination of one letter e.g. $y=-2z+5$ M1A1
	$=\begin{bmatrix}5\\-2\\1\end{bmatrix}$	A1F		Elimination of second letter e.g. $y=\dfrac{7-2x}{5}$ A1
	Equation of line is $\dfrac{x-1}{5}=\dfrac{y-1}{-2}=\dfrac{z-2}{1}$	M1A1F	5	Combining the results $-2z+5=y=\dfrac{7-2x}{5}$ M1 Rearranging $\dfrac{z-5/2}{1}=\dfrac{y}{-2}=\dfrac{x-7/2}{5}$ A1
(c)	$\cos\theta=\dfrac{\pm(0,1,0)\bullet(5,-2,\ 1)}{\sqrt{5^2+(-2)^2+1^2}}$	M1A1F		ft incorrect $(5,-2,\ 1)$
	$\theta=68.6^\circ$	A1F	3	
	Total		**9**	

Q	Solution	Marks	Total	Comments
5 (a)	**AB** = $\begin{bmatrix} 3 & -1 & p \\ 0 & -5 & p \end{bmatrix}\begin{bmatrix} p & -1 \\ -2 & 0 \\ -3 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 3p-3 \\ 10-3p & 3p \end{bmatrix}$	M1 A2,1,0	3	The order of the matrices must be correct for M1 Allow the M1 for two correctly positioned elements
(b)(i)	det $\mathbf{AB} = 6p + (3p-10)(3p-3)$	M1A1F		
	$=3\ (3p^2 - 11p + 10)$	A1F		or $9p^2 - 33p + 30 = 0$
	$= 3(3p-5)(p-2)$			
	$= 0$ when $p = \frac{5}{3}$ or 2	A1	4	ft if factorises
(ii)	$p = \frac{5}{3}$ $\quad \mathbf{AB} = \begin{bmatrix} 2 & 2 \\ 5 & 5 \end{bmatrix}$ $\quad \mathbf{B}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}} = \begin{bmatrix} 2 & 5 \\ 2 & 5 \end{bmatrix}$	M1A1F		M1 for either $p = 5$ or $p = 2$
	$p = 2$ $\quad \mathbf{AB} = \begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}$ $\quad \mathbf{B}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}} = \begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$	A1F	3	
(iii)	det $\mathbf{AB} = 0$	E1	1	
	Total		**11**	

Q	Solution	Marks	Total	Comments
6 (a)	$\begin{vmatrix} 3 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{vmatrix} = 0$	M1A1		
	$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$			
	$\therefore 3x = 0$ $y + 2z = 0$	M1A1		
	eigenvector is $\begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$	A1F	5	OE
(b)(i)	$\begin{vmatrix} 4-\lambda & 0 & 0 \\ 0 & 2-\lambda & 2 \\ 0 & 1 & 3-\lambda \end{vmatrix}$			
	$= (4-\lambda)((2-\lambda)(3-\lambda)-2)$	M1A1		Allow whenever this line appears
	$= (4-\lambda)(\lambda-4)(\lambda-1)$	A1F		Provided quadratic factorises
	$\lambda = 4$	A1	4	
(ii)	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -2 & 2 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ -2y+2z \\ y-z \end{bmatrix}$	M1		Accept $\begin{bmatrix} p \\ q \\ q \end{bmatrix}$ substituted in and verified
	$y = z$, x any value	A1		
	eigenvector $\begin{bmatrix} p \\ q \\ q \end{bmatrix}$	A1	3	AG
(c)(i)	$x = 0, y = -2z$ $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 1 & 3 \end{bmatrix}\begin{bmatrix} 0 \\ -2t \\ t \end{bmatrix} = \begin{bmatrix} 0 \\ -2t \\ t \end{bmatrix}$			
	$\therefore$ point invariant	M1A1	2	
(ii)	$x = 0, y = z$ $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 1 & 3 \end{bmatrix}\begin{bmatrix} 0 \\ t \\ t \end{bmatrix} = 4\begin{bmatrix} 0 \\ t \\ t \end{bmatrix}$			
	$\therefore$ invariant line	M1A1	2	
	Total		**16**	
	Total		**60**	