General Certificate of Education
June 2004
Advanced Level Examination

MATHEMATICS (SPECIFICATION A) **MAP5**
Unit Pure 5

Friday 11 June 2004 Morning Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 20 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP5.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 (a) Find $\displaystyle\int \frac{4}{x(x+4)}\,\mathrm{d}x$. *(3 marks)*

(b) Determine whether either of the following integrals can be evaluated. Where possible, evaluate the integral.

(i) $\displaystyle\int_0^1 \frac{4}{x(x+4)}\,\mathrm{d}x$ *(2 marks)*

(ii) $\displaystyle\int_1^\infty \frac{4}{x(x+4)}\,\mathrm{d}x$ *(3 marks)*

2 Use the result

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$$

to find the value of k for which

$$\lim_{x\to 0}\left(\frac{1-\cos^k x}{x^2}\right) = 4.$$ *(4 marks)*

3 The function $y(x)$ satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where $$\mathrm{f}(x, y) = x^2 + y^2 - 3$$

and $$y(1) = 1.$$

(a) Use the Euler formula

$$y_{r+1} = y_r + h\mathrm{f}(x_r, y_r)$$

starting with $(x_0, y_0) = (1, 1)$ and with step interval h to show that

$$y_1 \approx 1 - h.$$ *(2 marks)*

(b) (i) Use the formula

$$y_{r+1} = y_{r-1} + 2h\mathrm{f}(x_r, y_r)$$

with the same step interval h to show that

$$y_2 \approx 1 - 2h + 4h^3.$$ *(4 marks)*

(ii) Use your answer for y_2 in part (b)(i) to find an estimate for $y(1.1)$. *(2 marks)*

4 A curve has polar equation

$$\frac{2}{r} = 1 + \cos\theta.$$

Find its Cartesian equation in the form $y^2 = \mathrm{f}(x)$. *(6 marks)*

TURN OVER FOR THE NEXT QUESTION

5 (a) Show that the integrating factor for the differential equation

$$\frac{dy}{dx} - \frac{y}{x+1} = x^2, \qquad x > -1,$$

is $\dfrac{1}{x+1}$. *(3 marks)*

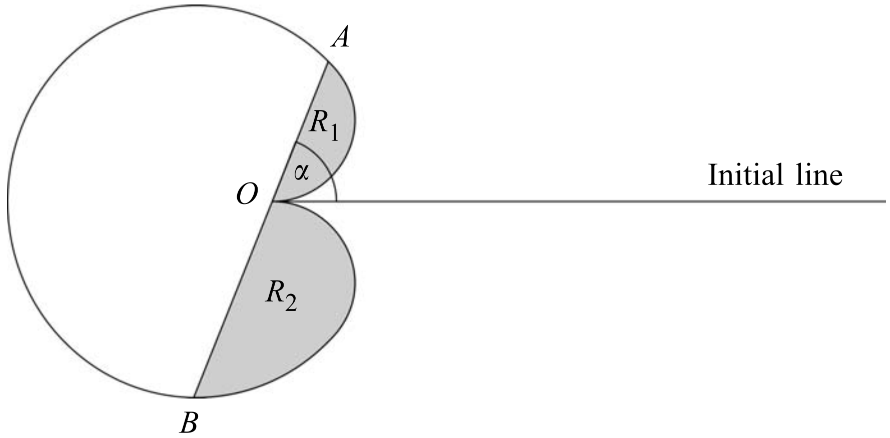
(b) Solve the differential equation

$$\frac{dy}{dx} - \frac{y}{x+1} = x^2, \qquad x > -1,$$

given that $y = 2$ when $x = 0$. *(6 marks)*

(c) Find $\lim\limits_{x \to -1} y$, giving a reason for your answer. *(1 mark)*

6 The diagram shows a sketch of a curve, whose polar equation is $r = 2(1 - \cos\theta)$, and a chord AB passing through the pole O and inclined at an angle α, $0 \leqslant \alpha \leqslant \frac{1}{2}\pi$, to the initial line.



(a) The areas of the regions enclosed between the curve and the lines OA and OB are denoted by R_1 and R_2 respectively. Show that

$$R_1 + R_2 = a\pi + b\sin\alpha,$$

where a and b are integers to be found. *(7 marks)*

(b) Show that the length of the chord AB is independent of α. *(3 marks)*

7 (a) Show that the substitution

$$u = \frac{dy}{dx} - ky,$$

where k is a constant, transforms the differential equation

$$\frac{d^2y}{dx^2} - 2k\frac{dy}{dx} + k^2y = 12xe^{kx}$$

into

$$\frac{du}{dx} - ku = 12xe^{kx}.$$ *(4 marks)*

(b) Find the general solution of

$$\frac{du}{dx} - ku = 12xe^{kx},$$

giving your answer in the form $u = f(x)$. *(5 marks)*

(c) Hence find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 2k\frac{dy}{dx} + k^2y = 12xe^{kx},$$

giving your answer in the form $y = g(x)$. *(5 marks)*

END OF QUESTIONS

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE