

General Certificate of Education
January 2005
Advanced Level Examination



MATHEMATICS (SPECIFICATION A)
Unit Pure 5

MAP5

Tuesday 18 January 2005 Afternoon Session

In addition to this paper you will require:

- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 20 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP5.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information

- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

- 1 (a) Write down the expansion of $\sin 2x$ in ascending powers of x up to and including the term in x^3 . (1 mark)

- (b) Hence find

$$\lim_{x \rightarrow 0} \frac{2x \cos x - \sin 2x}{x^3}. \quad (4 \text{ marks})$$

- 2 The function $y(x)$ satisfies the differential equation

$$\frac{dy}{dx} = x^2y + xy^2 + 1.$$

- (a) The use of the Euler formula

$$y_{r+1} = y_r + hf(x_r, y_r)$$

with $y_0 = y(1) = 1$ gives $y_1 = y(1+h) = 1.15$. Determine the value of h that has been used. (3 marks)

- (b) Show that, with this value of h , use of the same Euler formula gives $y_2 = 1.3328$ correct to four decimal places. (2 marks)

- (c) Use the formula

$$y_{r+1} = y_{r-1} + 2hf(x_r, y_r)$$

with the same value of h to find y_3 , giving your answer to three decimal places. (3 marks)

- 3 (a) Use integration by parts to show that

$$\int_k^1 \frac{\ln x}{\sqrt{x}} dx = 4(\sqrt{k} - 1) - 2\sqrt{k} \ln k,$$

where $0 < k < 1$. (5 marks)

- (b) Explain why

$$\int_0^1 \frac{\ln x}{\sqrt{x}} dx$$

exists, and find this integral. (2 marks)

- 4 (a) Obtain the roots of the equation

$$m^2 + 4m + 8 = 0,$$

giving your answers in the form $a \pm i b$. (2 marks)

- (b) Solve the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 8y = 8e^{-2x}$$

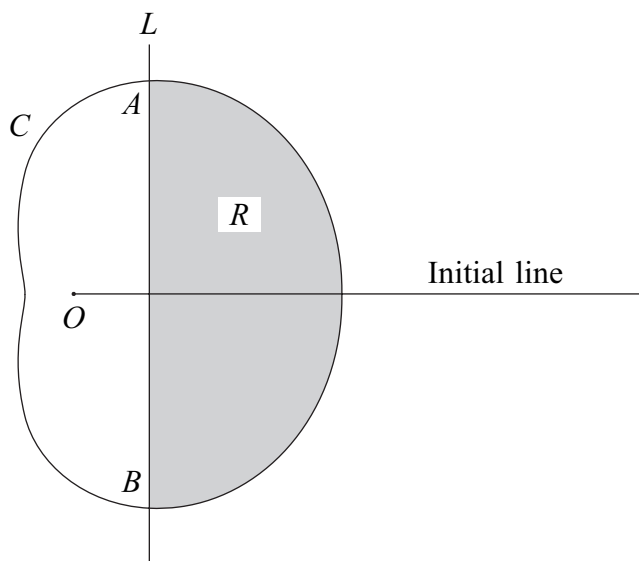
given that $y = 2$ and $\frac{dy}{dx} = 2$ when $x = 0$. (12 marks)

- 5 The diagram shows a sketch of the curve C and the line L , whose polar equations are

$$r = 3 + 2 \cos \theta$$

and $r \cos \theta = 2$

respectively.



- (a) The points of intersection of the curve C and the line L are A and B .

Show that $\theta = \frac{1}{3}\pi$ at A and find the polar coordinates of A and B . (6 marks)

- (b) Find the area of the shaded region R between the curve C and the line L . Give your answer in the form $p\pi + q\sqrt{3}$, where p and q are rational numbers. (9 marks)

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- 6 (a) Given that $y = \frac{1}{z}$, where z is a function of x , express $\frac{dy}{dx}$ in terms of z and $\frac{dz}{dx}$. (1 mark)
- (b) It is given that y satisfies the differential equation

$$x^2 \frac{dy}{dx} + yx = y^2, \quad x > 0.$$

Show that the substitution $y = \frac{1}{z}$ transforms this equation into the differential equation

$$\frac{dz}{dx} - \frac{z}{x} = -\frac{1}{x^2}. \quad (2 \text{ marks})$$

- (c) (i) Obtain the general solution of the differential equation

$$\frac{dz}{dx} - \frac{z}{x} = -\frac{1}{x^2}. \quad (5 \text{ marks})$$

- (ii) Hence obtain y in terms of x , given that $y = 2$ when $x = \frac{1}{2}$. (3 marks)

END OF QUESTIONS