General Certificate of Education
January 2004
Advanced Level Examination

MATHEMATICS (SPECIFICATION A) **MAP5**
Unit Pure 5

Friday 16 January 2004 Afternoon Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 20 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP5.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 The function $y(x)$ satisfies the differential equation

$$\frac{d^2y}{dx^2} = x^2 + y^2.$$

Given that $y = 2$ when $x = 1$ and that $y = 2.08$ when $x = 1.1$, use the formula

$$y_{r+1} \approx 2y_r - y_{r-1} + h^2 y_r''$$

with a step interval of 0.1 to estimate the value of $y(1.2)$. *(4 marks)*

2 (a) By means of a suitable substitution, or otherwise, evaluate

$$\int_0^a \frac{x}{\sqrt{1-x^2}}\,dx,$$

where $0 \leqslant a < 1$. *(4 marks)*

(b) Explain why the integral is improper when $a = 1$. *(1 mark)*

(c) Evaluate

$$\int_0^1 \frac{x}{\sqrt{1-x^2}}\,dx.$$

(2 marks)

3 (a) A curve C has polar equation

$$r = \mathrm{e}^{k\theta},$$

where $k \neq 0$ and $0 \leqslant \theta \leqslant \frac{1}{2}\pi$.

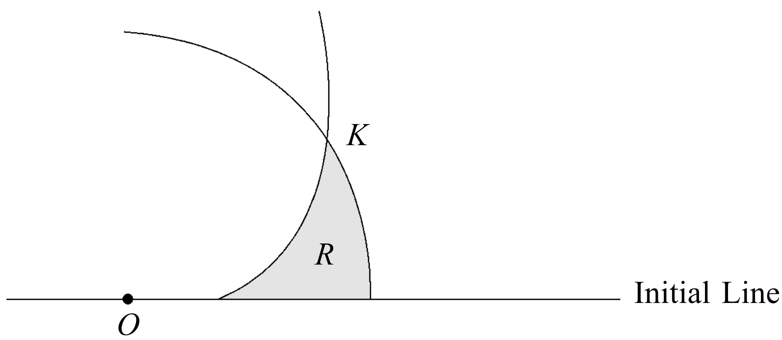
The points P and Q on C have polar coordinates (r_1, θ_1) and (r_2, θ_2) respectively, where $\theta_2 > \theta_1$.

Show that the area A bounded by C and the lines OP and OQ, where O is the pole, is given by

$$A = \frac{1}{4k}\left({r_2}^2 - {r_1}^2\right).$$

(4 marks)

(b) The diagram shows a sketch of part of the curve $r = \mathrm{e}^{\theta}$ and part of the circle $r = 2$.



(i) Find the polar coordinates of K, the point of intersection of the two curves. *(2 marks)*

(ii) Find the area of the shaded region R, between the curves and the initial line, giving your answer in the form $p \ln 2 + q$ where p and q are rational numbers. *(5 marks)*

TURN OVER FOR THE NEXT QUESTION

4 (a) (i) Use the approximation

$$\cos x \approx 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$$

to show that the first three non-zero terms in the expansion in ascending powers of x of $\dfrac{1}{\cos x}$ are

$$1 + \frac{x^2}{2} + \frac{5x^4}{24}.$$

(5 marks)

(ii) By writing $\tan x = \dfrac{\sin x}{\cos x}$, find the first three non-zero terms in the expansion in ascending powers of x of $\tan x$. *(4 marks)*

(b) Find

$$\lim_{x \to 0} \left(\frac{\tan 2x - 2x}{\tan x - x} \right).$$

(4 marks)

5 (a) Given that $y = ax^2 + bx$ is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} = x,$$

find the values of a and b. *(4 marks)*

(b) Hence, or otherwise, solve the differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} = x,$$

given that $y = 1$ and $\dfrac{dy}{dx} = 3$ when $x = 0$. *(7 marks)*

6 (a) The function $y(x)$ satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y),$$

where $$\mathrm{f}(x, y) = \frac{x^3 + y^3}{xy^2}$$

and $$y(1) = 1.$$

Use the formula

$$y_{r+1} \approx y_r + \frac{1}{2}(k_1 + k_2)$$

where $$k_1 = h\,\mathrm{f}(x_r, y_r)$$

and $$k_2 = h\,\mathrm{f}(x_r + h, y_r + k_1),$$

with $h = 0.1$ to calculate a value for $y(1.1)$, giving your answer to four decimal places.
(5 marks)

(b) (i) Use the substitutions $y = ux$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = u + x\frac{\mathrm{d}u}{\mathrm{d}x}$ to show that the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x^3 + y^3}{xy^2}$$

transforms to

$$x\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{u^2}.$$ *(3 marks)*

(ii) Solve the differential equation

$$x\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{u^2}$$

and hence express y in terms of x, given that $y = 1$ when $x = 1$. *(5 marks)*

(iii) Use your solution to part (b)(ii) to find the value of $y(1.1)$, giving your answer to four decimal places. *(1 mark)*

END OF QUESTIONS