

# GCE 2005

## *January Series*



# Mark Scheme

## Mathematics A

*(MAP5)*

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*Dr Michael Cresswell Director General*

## Key to Mark Scheme

<b>M</b> .....	mark is for .....	method
<b>m</b> .....	mark is dependent on one or more M marks and is for .....	method
<b>A</b> .....	mark is dependent on M or m marks and is for .....	accuracy
<b>B</b> .....	mark is independent of M or m marks and is for .....	method and accuracy
<b>E</b> .....	mark is for .....	explanation
<b>✓ or ft or F</b> .....	follow through from previous	incorrect result
<b>CAO</b> .....	correct answer only	
<b>AWFW</b> .....	anything which falls within	
<b>AWRT</b> .....	anything which rounds to	
<b>AG</b> .....	answer given	
<b>SC</b> .....	special case	
<b>OE</b> .....	or equivalent	
<b>A2,1</b> .....	.2 or 1 (or 0) accuracy marks	
<b>-x EE</b> .....	deduct x marks for each error	
<b>NMS</b> .....	no method shown	
<b>PI</b> .....	possibly implied	
<b>SCA</b> .....	substantially correct approach	
<b>c</b> .....	candidate	
<b>SF</b> .....	significant figure(s)	
<b>DP</b> .....	decimal place(s)	

## Abbreviations used in Marking

<b>MC – x</b> .....	deducted x marks for mis-copy
<b>MR – x</b> .....	deducted x marks for mis-read
<b>ISW</b> .....	ignored subsequent working
<b>BOD</b> .....	given benefit of doubt
<b>WR</b> .....	work replaced by candidate
<b>FB</b> .....	formulae booklet

## Application of Mark Scheme

### **No method shown:**

Correct answer without working .....	mark as in scheme
Incorrect answer without working .....	zero marks unless specified otherwise

### **More than one method/choice of solution:**

2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only

### **Crossed out work**

do not mark unless it has not been replaced

**Alternative solution** using a correct or partially correct method

award method and accuracy marks as appropriate

## MAP5

Q	Solution	Marks	Total	Comments
1(a)	$\sin 2x = 2x - \frac{8x^3}{6}$	B1	1	Ignore extra terms
(b)	Use of $\left(1 - \frac{x^2}{2}\right)$ and $\left(2x - \frac{8x^3}{6}\right)$	M1		
	$L = \lim_{x \rightarrow 0} \frac{2x\left(1 - \frac{x^2}{2}\right) - \left(2x - \frac{8x^3}{6}\right) + 0(x^5)}{x^3}$	A1F		Condone $0(x^5)$ missing
	$= \lim_{x \rightarrow 0} \frac{-x^3 + \frac{8x^3}{6} + 0(x^5)}{x^3}$	A1F		
	$= \frac{1}{3}$	A1F	4	
<b>Total</b>			<b>5</b>	
2(a)	$1.15 = 1 + h(1 + 1 + 1)$ $h = 0.05$	M1A1 A1	3	
(b)	$y_2 = 1.15 + .05f(1.05, 1.15)$ $= 1.3328$	M1 A1	2	Applied AG
(c)	$y_3 = 1.15 + 0.1f(1.1, 1.3328)$ $= 1.607$	M1A1 A1	3	2 clear errors M0
<b>Total</b>			<b>8</b>	
3(a)	$I = \left[2\sqrt{x} \ln x\right]_k^1 - \int_k^1 2\sqrt{x} \frac{1}{x} dx$ $\left[-4\sqrt{x}\right]_k^1$ $= 4(\sqrt{k} - 1) - 2\sqrt{k} \ln k$	M1A1 A1  A1F  A1	5	AG
(b)	Exists since $\sqrt{k} \ln k \rightarrow 0$ as $k \rightarrow 0$ value is $-4$	E1 A1F	2	Clear explanation If M0 earlier, allow B1 for $-4$
<b>Total</b>			<b>7</b>	

**MAP5 (cont)**

Q	Solution	Marks	Total	Comments	
<b>4(a)</b>	$m = \frac{1}{2}(-4 \pm \sqrt{16 - 32})$ $= -2 \pm 2i$	M1	2		
		A1			
<b>(b)</b>	C.F. $y = e^{-2x} (A \cos 2x + B \sin 2x)$	M1A1F	12	Provided roots are non-real	
	P.I. Try $y = Pe^{-2x}$	M1		If P.I. = $Ax^k e^{-2x}$ ( $k \neq 0$ ) then M0	
	$\frac{dy}{dx} = -2Pe^{-2x}; \quad \frac{d^2y}{dx^2} = 4Pe^{-2x}$	A1		A0	
	Sub into D.E.	M1		M0	
	$4P = 8, P = 2$	A1F		A0	
	G.S. $y = e^{-2x} (A \cos 2x + B \sin 2x + 2)$	B1F		i.e. C.F. + P.I.	B1
	$y = 2, x = 0 \quad \therefore A = 0$	B1F		B0	
	$\frac{dy}{dx} = -2Be^{-2x} \sin 2x + 2Be^{-2x} \cos 2x - 4e^{-2x}$	M1A1F		Then follow through last 4 marks	
	$2 = 2B - 4 \quad B = 3$	A1F			
	G.S. $y = (3 \sin^2 x + 2)e^{-2x}$	A1F			
<b>Total</b>			<b>14</b>		

## MAP5 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$\frac{2}{\cos \theta} = 3 + 2 \cos \theta$	M1	6	or corresponding results in $r$
	$2 \cos^2 \theta + 3 \cos \theta - 2 = 0$ $(2 \cos \theta - 1)(\cos \theta + 2) = 0$ $\cos \theta = \frac{1}{2}, \cos \theta \neq -2$ at $A, \theta = \frac{\pi}{3}$ $A$ and $B$ are $\left(4, \pm \frac{1}{3}\pi\right)$	A1 m1 A1 A1 A1		Accept $\left(4, \frac{5\pi}{3}\right)$
(b)	Area $S$ bounded by $C, OA, OB$			
	$S = \frac{1}{2} \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} (3 + 2 \cos \theta)^2 d\theta$	M1		Ignore limits here
	$= \int_0^{\frac{\pi}{3}} (9 + 12 \cos \theta + 4 \cos^2 \theta) d\theta$	A1		
	Use of $\cos 2\theta = 2 \cos^2 \theta - 1$	m1		For an attempt to express $\cos^2 \theta$ in terms of $\cos 2\theta$
	$S = \int_0^{\frac{\pi}{3}} (11 + 12 \cos \theta + 2 \cos 2\theta) d\theta$	A1F		
	$= [11\theta + 12 \sin \theta + \sin 2\theta]_0^{\frac{\pi}{3}}$	A1F		
	$= \frac{11\pi}{3} + \frac{13\sqrt{3}}{2}$	A1F		Correct limits needed here
Area of $\triangle OAB = \frac{1}{2} 4^2 \sqrt{\frac{3}{2}} = 4\sqrt{3}$	M1A1		Allow M1 for $[2 \tan \theta]_{-\frac{\pi}{3}}^{\frac{\pi}{3}}$	
Area of $R = \frac{11\pi}{3} + \frac{13\sqrt{3}}{2} - 4\sqrt{3}$ $= \frac{11\pi}{3} + \frac{5\sqrt{3}}{2}$	A1F	9		
	<b>Total</b>		<b>15</b>	

**MAP5 (cont)**

Q	Solution	Marks	Total	Comments
<b>6(a)</b>	$y = \frac{1}{z}, \frac{dy}{dx} = -\frac{1}{z^2} \frac{dz}{dx}$	B1	1	
<b>(b)</b>	$x^2 \left( -\frac{1}{z^2} \frac{dz}{dx} \right) + \frac{1}{z} x = \frac{1}{z^2}$	M1		
	$\frac{dz}{dx} - \frac{z}{x} = -\frac{1}{x^2}$	A1	2	
<b>(c)(i)</b>	I.F = $e^{-\int \frac{1}{x} dx} = \frac{1}{x}$	M1A1		
	$\frac{d}{dx} \left( \frac{z}{x} \right) = -\frac{1}{x^3}$	M1A1F		
	$\frac{z}{x} = \frac{1}{2x^2} + c$	A1F	5	
<b>(ii)</b>	$x = \frac{1}{2}, z = \frac{1}{2} \quad c = -1$	A1F		
	$z = \frac{x(1-2x^2)}{2x^2}$	A1F		
	$y = \frac{2x}{1-2x^2}$	A1F	3	Accept $\left( \frac{1}{2x} - x \right)^{-1}$ OE
	<b>Total</b>		<b>11</b>	
	<b>Total</b>		<b>60</b>	