ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark scheme
January 2004

GCE

Mathematics A

Unit MAP5

Key to mark scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m mark and is for	accuracy
B	mark is independent of M or m marks and is for	method and accuracy
E	mark is for	explanation
√ or ft or F		follow through from previous incorrect result
CAO		correct answer only
AWFW		anything which falls within
AWRT		anything which rounds to
AG		answer given
SC		special case
OE		or equivalent
A2,1		2 or 1 (or 0) accuracy marks
– *x* EE		Deduct *x* marks for each error
NMS		No method shown
PI		Perhaps implied
c		Candidate

Abbreviations used in marking

MC – *x*	deducted *x* marks for miscopy
MR – *x*	deducted *x* marks for misread
ISW	ignored subsequent working
BOD	gave benefit of doubt
WR	work replaced by candidate

Application of mark scheme

Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise

Award method and accuracy marks as appropriate to an alternative solution using a correct method or partially correct method.

Q	Solution	Marks	Total	Comments
1	$y(1.2) \approx 2y(1.1) - y(1) + 0.1^2\left(1.1^2 + 2.08^2\right)$	M1A1		
	$= 2 \times 2.08 - 2 + 0.055364$	A1F		
	$= 2.22$ (2.215364)	A1F	4	AWRT
	Total		**4**	
2 (a)	$u = 1 - x^2$ $\quad$ $\mathrm{d}u = -2x\mathrm{d}x$ or $x = \sin\theta$ $\quad$ $\mathrm{d}x = \cos\theta\,\mathrm{d}\theta$	M1		
	$\mathrm{I} = \int \frac{-\mathrm{d}u}{2u^{\frac{1}{2}}}$ or $\mathrm{I} = \int \sin\theta\,\mathrm{d}\theta$	A1		Limits not needed here
	$= \left[-u^{\frac{1}{2}}\right]$ or $\left[-\cos\theta\right]$	A1F		Limits not needed
	$= 1 - \left(1 - a^2\right)^{\frac{1}{2}}$	A1F	4	ft provided $p\left(1 - \left(1 - a^2\right)^{\frac{1}{2}}\right)$ where p is an integer
(b)	When $a = 1$, denominator is zero	E1	1	
(c)	$a = 1$, $I = 1$	M1A1F	2	
	Total		**7**	

Q	Solution	Marks	Total	Comments
3 (a)	$A = \frac{1}{2}\int_{\theta_1}^{\theta_2} e^{2k\theta} d\theta$	M1A1		A1 for $e^{2k\theta}$
	$= \frac{1}{4k}\left[e^{2k\theta}\right]_{\theta_1}^{\theta_2}$	A1		
	$= \frac{1}{4k}\left[e^{2k\theta_2} - e^{2k\theta_1}\right]$			
	$= \frac{1}{4k}\left(r_2^2 - r_1^2\right)$	A1	4	AG
(b)(i)	at $K, e^{\theta} = 2$	M1		
	$\theta = \ln 2$			
	K is (2, ln 2)	A1	2	Accept $(2, 0.69(3))$
(ii)	Area of sector of circle is $\frac{1}{2} \times 2^2 \ln 2$			
	$= 2\ln 2$	B1		
	Area under curve by (a) is $\frac{1}{4}\left(2^2 - 1^2\right)$	M1		
	$= \frac{3}{4}$	A1		
	Shaded area = $2\ln 2 - \frac{3}{4}$	M1A1F	5	M0 if added or subtracted the wrong way round ft simple slips
	Total		**11**	

Q	Solution	Marks	Total	Comments
4 (a)(i)	$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{24}$	B1		Simplification of factorials continued sensibly
	$\frac{1}{\cos x} = \left(1 - \frac{x^2}{2} + \frac{x^4}{24}\right)^{-1}$	M1		
	$= 1 + \left(\frac{x^2}{2} - \frac{x^4}{24}\right) + \left(\frac{x^2}{2} - \frac{x^4}{24}\right)^2$	M1		
	$= 1 + \frac{x^2}{2}$	A1		AG
	$+ \frac{5x^4}{24}$	A1	5	AG
(ii)	$\tan x = \left(x - \frac{x^3}{6} + \frac{x^5}{120}\right)\left(1 + \frac{x^2}{2} + \frac{5x^4}{24}\right)$	M1A1		
	$= x + \frac{x^3}{3}$	A1F		Incorrect sin series
	$+ \frac{2x^5}{15}$ or $\frac{16x^5}{120}$	A1F	4	
(b)	$\lim\left(\frac{\tan 2x - 2x}{\tan x - x}\right) = \frac{2x + \frac{8x^3}{3} + \frac{64x^5}{15} - 2x}{x + \frac{x^3}{3} + \frac{2x^5}{15} - x}$	M1A1F		
	$= \frac{\frac{8}{3} + \mathrm{O}(x^2)}{\frac{1}{3} + \mathrm{O}(x^2)}$	A1F		
	$= 8$	A1F	4	
	Total		**13**	

Q	Solution	Marks	Total	Comments
5 (a)	$y = ax^2 + bx$			
	$\left.\begin{array}{l}\frac{dy}{dx} = 2ax + b \\ \frac{d^2y}{dx^2} = 2a\end{array}\right\}$	M1A1		
	$2a + 2ax + b = x$	m1		
	$a = \frac{1}{2}, b = -1$	A1F	4	
(b)	Auxiliary equation $m^2 + m = 0$	M1		
	$m = 0$ or -1	A1		
	CF : $y = A + Be^{-x}$	A1F		Provided m's are real
	GS : $y = A + Be^{-x} + \frac{1}{2}x^2 - x$	A1F		
	$\frac{dy}{dx} = -Be^{-x} + x - 1$	M1		Provided the GS is differentiated
	$A = 5 \quad B = 4$	A1A1F	7	
	(GS $y = 5 - 4e^{-x} + \frac{1}{2}x^2 - x$)			
	Total		**11**	

Q	Solution	Marks	Total	Comments
6 (a)	$k_1 = 0.1\left(\frac{1^3+1^3}{1\times 1^2}\right) = 0.2$	B1		
	$k_2 = 0.1\left(\frac{1.1^3+1.2^3}{1.1\times 1.2^2}\right)$	M1A1		
	$y(1.1) = 1+\frac{1}{2}(0.2+0.193118686)$	M1		
	= 1.1965 (59343)			
	= 1.1966 (4dp)	A1F	5	
(b)(i)	$u + x\frac{du}{dx} = \frac{x^3+u^3x^3}{u^2x^3} = \frac{1+u^3}{u^2}$	M1A1		
	$x\frac{du}{dx} = \frac{1}{u^2} + u - u = \frac{1}{u^2}$	A1	3	AG
(ii)	$\int u^2 du = \int \frac{1}{x} dx$	M1		Separation of variables
	$\frac{u^3}{3} = \ln x + c$	A1		
	$\frac{y^3}{3x^3} = \ln x + c$	m1		
	$c = \frac{1}{3}$	A1		
	$y = x(3\ln x + 1)^{\frac{1}{3}}$	A1	5	Must be y not y^3
(iii)	$y(1.1) = 1.1961(85468)$			
	=1.1962 (4dp)	B1	1	CAO
	Total		**14**	
	Total		**60**	