General Certificate of Education
June 2004
Advanced Level Examination

MATHEMATICS (SPECIFICATION A) **MAP4**
Unit Pure 4

Wednesday 16 June 2004 Afternoon Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 20 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP4.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.
- Sheets of graph paper are available on request.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 (a) Show that $(3-\mathrm{i})^2 = 8 - 6\mathrm{i}$. *(1 mark)*

(b) The quadratic equation

$$az^2 + bz + 10\mathrm{i} = 0,$$

where a and b are real, has a root $3 - \mathrm{i}$.

(i) Show that $a = 3$ and find the value of b. *(6 marks)*

(ii) Determine the other root of the quadratic equation, giving your answer in the form $p + \mathrm{i}q$. *(3 marks)*

2 (a) Show that

$$\frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)} = \frac{2}{r(r+1)(r+2)}.$$ *(2 marks)*

(b) Hence find the sum of the series

$$\frac{1}{1\times2\times3} + \frac{1}{2\times3\times4} + \frac{1}{3\times4\times5} + \ldots + \frac{1}{30\times31\times32},$$

giving your answer as a rational number. *(5 marks)*

3 (a) Sketch on one Argand diagram:

(i) the locus of points satisfying $|z-\mathrm{i}| = |z-2|$; *(2 marks)*

(ii) the locus of points satisfying $\arg(z-\mathrm{i}) = \frac{1}{4}\pi$. *(2 marks)*

(b) Shade on your diagram the region in which

$$|z-\mathrm{i}| \leqslant |z-2| \quad \text{and} \quad -\tfrac{1}{2}\pi \leqslant \arg(z-\mathrm{i}) \leqslant \tfrac{1}{4}\pi.$$ *(3 marks)*

4 The roots of the cubic equation

$$x^3 + 9x^2 + 27x + 35 = 0$$

are α, β and γ.

(a) Use the substitution $x = y - 3$ to show that the cubic equation which has roots $\alpha + 3$, $\beta + 3$ and $\gamma + 3$ is

$$y^3 + 8 = 0\,.$$ *(6 marks)*

(b) (i) Find the **three** roots of $y^3 + 8 = 0$, giving each root in the form $a + \mathrm{i}b$. *(3 marks)*

(ii) Hence, or otherwise, find the roots of the equation

$$x^3 + 9x^2 + 27x + 35 = 0.$$ *(2 marks)*

5 (a) Find the constants A and B in the identity

$$\left(z^2 - \frac{1}{z^2}\right)^3 \equiv A\left(z^2 - \frac{1}{z^2}\right) + B\left(z^6 - \frac{1}{z^6}\right).$$ *(3 marks)*

(b) (i) Use the result

$$z^n - \frac{1}{z^n} = 2\mathrm{i}\sin n\theta,$$

where $z = \cos\theta + \mathrm{i}\sin\theta$,

to show that

$$\sin^3 2\theta = \tfrac{3}{4}\sin 2\theta - \tfrac{1}{4}\sin 6\theta.$$ *(4 marks)*

(ii) Hence, or otherwise, show that

$$\int_0^{\frac{1}{4}\pi} \sin^3 2\theta \,\mathrm{d}\theta = \tfrac{1}{3}.$$ *(3 marks)*

6 (a) Show that

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(2\tan^{-1}\mathrm{e}^{t}\right)=\operatorname{sech} t\,. \qquad \textit{(5 marks)}$$

(b) A curve C is given parametrically by

$$x=2+\tanh t,\quad y=2-\operatorname{sech} t\,.$$

(i) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}t}=\operatorname{sech} t\tanh t. \qquad \textit{(3 marks)}$$

(ii) Express $\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2$ in terms of $\operatorname{sech} t$. *(3 marks)*

(c) The arc of the curve C between $t=0$ and $t=1$ is rotated through 2π radians about the x-axis.

(i) Show that S, the surface area generated, is given by

$$S=2\pi\int_0^1(2-\operatorname{sech} t)\operatorname{sech} t\,\mathrm{d}t\,. \qquad \textit{(1 mark)}$$

(ii) Hence show that

$$S=2\pi(4\tan^{-1}\mathrm{e}-\tanh 1-\pi). \qquad \textit{(3 marks)}$$

END OF QUESTIONS